Math 112 (Calculus I)
Final Exam
Dec 18, 7:00 p.m.

Instructions:

• Work on scratch paper will not be graded.
• For questions 11 to 19, show all your work in the space provided.. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
• Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
• Simplify your answers. Expressions such as ln(1), e^0, sin(\pi/2), etc. must be simplified for full credit.
• Calculators are not allowed.

For Instructor use only.

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Short Answer Fill in the blank with the appropriate answer.

1. (10 points)

a) \( \frac{d}{dx} \ln(\tan x) = \) _________________

b) Use the linearization of \( f(x) = x^{1/3} \) at \( a = 8 \) to approximate \( 9^{1/3} \). _________________

c) If \( f'(x) = x^3 \) and \( f(0) = 5 \) then \( f(x) = \) _________________.

d) If \( f(x) = e^{2x} \) then \( f''(x) = \) _________________.

e) The Mean Value Theorem says that if \( f \) is a _________________ function on \( [a, b] \) which is also _________________ on \( (a, b) \) then there is a \( c \in (a, b) \) with _________________.

f) Circle the correct answer in both cases: If \( f' \) is positive and increasing, then \( f \) is (increasing / decreasing) and (concave up / concave down).

g) \( \lim_{x \to \infty} \frac{x^3 + 5}{2x^4 + 3x + 2} = \) _________________.
Multiple Choice. In the grid below fill in the square corresponding to each correct answer.

2. If \( \sin(xy) = \frac{x - 1}{y} \), what is \( y' \) at \((1, \pi)\)?

   a) \( \pi \)  
   b) \( \pi^2 - 1 \)  
   c) \( \pi - \frac{1}{\pi} \)
   d) \( -\frac{\pi^2 - 1}{\pi} \)  
   e) \( -1 - \frac{1}{\pi} \)  
   f) \( -\frac{1}{\pi} \)

3. Which of the following has a removable discontinuity at \( x = 1 \)?

   a) \( \frac{x^2 - 1}{x + 2} \)  
   b) \( \ln|x^2 - 1| \)  
   c) \( \frac{x^2 + 2}{x - 1} \)
   d) \( \frac{x^2 - 3x + 2}{x^2 - 1} \)  
   e) \( \tan\frac{\pi x}{2} \)  
   f) \( \sin\frac{1}{1 - x} \)

4. If the position of a particle is given by \( \frac{t^2 - t}{t - 1} \), at what value(s) of \( t \) is the velocity 0?

   a) 1  
   b) \( 1 \pm \sqrt{2} \)  
   c) \( 2 \pm \sqrt{2} \)
   d) \( 1 - 2\sqrt{2} \)  
   e) 0  
   f) The velocity is never 0.
5. If \( f(x) \) is a differentiable function, which of the following is not always true?

    a) \( \int_a^x f(t)dt = f(x) - f(a) \)  
    b) \( \frac{d}{dx} \int_a^x f(t)dt = f(x) \)  
    c) \( \int f'(x)dx = f(x) + C \)  
    d) \( \int_a^x F(t)dt = F'(x) - F'(a) \)  
    e) If \( F'(x) = f(x) \) then \( \int_a^x f(t)dt = F(x) - F(a) \)

6. A 20 ft ladder is placed against a wall. If the top of the ladder is dropping at a rate of 5 ft/min, how fast is the bottom of the ladder moving away from the wall (in feet per minute) when the top of the ladder is 12 ft high?

    a) -32  
    b) -160  
    c) 160  
    d) \( \frac{15}{2} \)  
    e) \( \frac{15}{4} \)  
    f) \( \frac{15}{16} \)

7. If \( h(x) = x \cosh(x^2 - 4) \), what is \( h''(2) \)?

    a) 0  
    b) 4  
    c) 8  
    d) 16  
    e) 32  
    f) 48

8. Let \( F(x) = \int_3^{2x^2} \frac{\sin(t)}{t^3 + 1} dt \) be defined for \( x > -1 \). Find \( F'(x) \).

    a) \( \frac{\sin(2x^2)}{8x^3 + 1} \)  
    b) \( \frac{4x \sin(2x^2)}{8x^6 + 1} \)  
    c) \( \frac{\sin(2x^2)}{8x^3 + 1} - \frac{\sin(18)}{216} \)  
    d) \( \frac{4x \sin(2x^2)}{8x^3 + 1} - \frac{12 \sin(3)}{28} \)  
    e) \( \frac{\sin(x)}{x^3 + 1} \)  
    f) \( \frac{\sin(x)}{x^3 + 1} - \frac{\sin(3)}{28} \)

9. If \( f(x) \) is a continuous function on the interval \( [4,5] \) and if \( f(4) = 3 \) and \( f(5) = 1 \), then which theorem guarantees that there is some value \( c \in [4,5] \) such that \( f(c) = 2 \)?

    a) Mean value Theorem,  
    b) Extreme Value Theorem,  
    c) Intermediate Value Theorem  
    d) The Fundamental Theorem of Calculus  
    e) Rolle’s Theorem  
    f) No theorem guarantees this because it is false.

10. Use one iteration of Newton’s method, beginning with \( x_1 = 1/2 \) to approximate the positive root of the equation \( x^2 + 2x - 1 = 0 \). (Note that the root is \( \sqrt{2} - 1 \)).

    a) \( \frac{1}{12} \)  
    b) \( \frac{5}{12} \)  
    c) 0  
    d) \( \frac{1}{2} \)  
    e) \( \frac{7}{12} \)  
    f) \( x_2 \) is undefined.
Free Response. For problems 11 - 19, write your answers in the space provided. Use the back of the page if needed, indicating that fact. Neatly show all work.

11. (7 points) State the definition of the derivative of $f(t)$, and use the definition to find the derivative of $f(t) = 2t^2 + 1$.

12. (7 points) Find the equation of the tangent line to $y = x\sqrt{x + 1}$ at the point (3, 6).

13. (7 points) Find the integral $\int_1^3 \frac{x}{2x^2 + 1} \, dx$. 
14. (7 points) \( \frac{d}{dx} x^{\cos x} = \)

15. (7 points) Find the absolute minimum and maximum of \( f(x) = x^2 e^{-x^2/2} \) on \([0, 4]\).

16. (7 points) Find \( \lim_{x \to 0} \frac{(1 - x)e^x - 1}{x^2} \).
17. (7 points) An open-at-the-top vertical tank has horizontal cross-section of an equilateral triangle, and volume of 4000 cubic feet. Find the dimensions that minimize the surface area.

(Hint: the height of an equilateral triangle is $\frac{\sqrt{3}}{2}$ times the length of a side.)

![Diagram of an equilateral triangle with height labeled h and side labeled s.]

18. (7 points) Given the following graph for $f(x)$, sketch $f'$. 

![Graph of a function f(x) with x-axis from -2 to 2 and y-axis from -1 to 3. There are local maxima at x = -1 and x = 1, and a minimum at x = 0.]
19. (7 points) Prove that \( \lim_{{x \to a}} 2x = 2a \), using the \( \epsilon, \delta \) definition of limit.