Math 119
Pretest Review – Questions

Linear Equations
Find the slope of the line passing through the given points:
1. (2, -5); (0, 2)
2. (3, -4); (3, 0)
3. (-1, -2); (1, 4)
4. Find the equation of the line with slope 2 that passes through (-2, -3). Write your answer in slope-intercept form.
5. Find the general form of the equation of the line that passes through (-1,6) and is parallel to the graph of $2x + 3y = 4 = 0$.
6. Find the general form of the equation of the line that passes through (4,-2) and is parallel to the graph of $x - y - 3 = 0$.

Quadratic Equations
Rewrite the following equations in general quadratic form:
7. $\frac{1}{x + 1} - \frac{3}{x - 2} = 5$
8. $\frac{3x - 2}{5} = x^2 + 1$
9. $x + 1 = \frac{x}{x + 2}$

Complete the square for the following functions:
10. $2x^2 - 8x + 5$
11. $-x^2 - 4x - 7$
12. $2x^2 - 4x$

Factoring
Factor:
13. $4x^3y - 16x^2y - 28y$
14. $x^3 + 2x^2 - 7x - 14$
15. $5xy^2 + 5y^2 + 3ax + 3a$ (Group in pairs)
16. $(x + 3)^3 - 1$
17. $a^3 - 64$
18. $9y^2 - 64$

Finding Zeros/Roots
19. Use Descarte’s Rule of Signs to determine the possible number of positive and negative zeros: $f(x) = x^3 + 1$
20. Given $f(x) = x^4 - 3x^3 + x^2 - 6x - 5$, determine the possible number of negative zeros.
21. List the possible rational zeros of the function: $f(x) = 3x^5 + 2x^2 - 3x + 2$
22. List the possible rational zeros of the function: $f(x) = 4x^3 + 3x^2 - 5x + 6$
23. Use the fact that $i$ is a zero of $f$ to find the remaining zeros: $f(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$
24. Find all of the zeros of the function: $f(x) = x^4 + 25x^2 + 144$
25. Find all the real zeros of the polynomial function: $g(t) = t^3 + 3t^2 - 16t - 48$
26. Use the Intermediate Value Theorem to estimate the real zero in the interval $[1, 2]$:
   
27. Use the Intermediate Value Theorem to estimate the real zero in the interval $[3, 4]$:
   
28. Use the Intermediate Value Theorem to estimate the real zero in the interval $[0, 1]$:
Long and Synthetic Division

29. Find all of the real roots:
\[ 2x^3 + 5x^2 - x - 6 = 0 \]

30. Find all of the real zeros of the function:
\[ f(x) = 2x^3 - 7x^2 + 7x - 2 \]

31. Write as a product of linear factors:
\[ f(x) = x^4 - 6x^3 - 4x^2 + 40x + 32 \]

32. Simplify the rational expression:
\[ \frac{2x^4 - x^3 - 4x^2 + x - 3}{2x + 3} \]

33. Simplify the rational function:
\[ f(x) = \frac{x^3 + 4x^2 - 3x + 10}{x + 5} \]

34. Simplify:
\[ \frac{x^4 + 3x^3 - 3x^2 - 12x - 4}{x^2 + 3x + 1} \]

Absolute Value

Solve the following equations for x:

35. \[ |x - 3| = 2 \]

36. \[ |x| = 5 \]

37. \[ |x + 5| = -1 \]

Pythagorean Theorem

For questions 41-42, consider a right triangle with sides of length a, b, and c:

41. If the lengths of sides a and b are six and eight inches respectively, how long is side c?

42. What is the length of side b in terms of the sides a and c?

Distance Formula

Find the distance between the given points:

43. (2, 5); (-1,9)

44. (-1, 3); (5,1)

45. (10, -3); (3, 0)

Simplifying Expressions

Simplify:

46. \[ \sqrt[4]{\frac{16x^4}{7y}} \] (Assume all variables represent positive real numbers)

47. \[ (\sqrt{x + 2} + 2)^2 \]

48. \[ \frac{x + 1}{x - 1} - \frac{3}{x + 2} \]

49. \[ \frac{27x^3y(x - 2)}{3xy^2(x^2 - 4)} \]

50. \[ \sqrt{x^3 - 3x + 2} \]

51. \[ \frac{6\sqrt{x^3 - 3x + 2}}{\sqrt{3x^2 + 3x - 6}} \]
Rationalize the denominator and simplify:

52. \( \frac{1}{\sqrt{50}} \)
53. \( \frac{2}{\sqrt{2}} \)
54. \( \frac{12}{\sqrt{6}} \)

Solving Equations
Solve for \( x \) or \( y \):

55. \((x^2 - 2x + 5)^{\frac{2}{3}} = 4\)
56. \( \frac{1}{x} - \frac{1}{x + 1} = 1 \)
57. \( \frac{x}{x^2 - 9} + \frac{2}{x + 3} = 3 \)
58. \( \sqrt{2x + 9} = x + 5 \)
59. \( 3 + \sqrt{x - 1} = x \)
60. \( x - 1 = \sqrt{1 - 5x} \)
61. \( \frac{y}{y - 3} + 3 = \frac{3}{y - 3} \)
62. \( \frac{y}{y - 3} + 3 = \frac{-1}{y - 3} \)
63. \( \frac{y}{y - 3} + 1 = \frac{-1}{y - 3} \)
64. \( x - 2 = \sqrt{x} \)
65. \( 2\sqrt{x} = x \)
66. \( 2\sqrt{x} + 3 = x + 3 \)

Inequalities
Solve each inequality:

67. \( x > 3x - 2 \)
68. \( 3x < 4x + 1 \)
69. \( x - 1 \leq 3x + 2 \)
70. \( \frac{1 - x}{x^2 - x - 6} > 0 \)
71. \( \frac{x + 1}{x - 3} \leq 0 \)
72. \( \frac{x^2 - 1}{x} \geq 0 \)
73. \( x^2 + 1 > x + 3 \)
74. \( x^3 - 2x^2 + 6x + 3 \leq x^2 + 10x + 3 \)
75. \( x^2 - 4x + 4 < x^3 - 6x^2 + 11x - 5 \)

Solve each absolute value inequality:

76. \( |x - 3| < 2 \)
77. \( |x + 5| \leq 0 \)
78. \( |x + 2| \geq 3 \)

Logarithmic, Exponential and Logistic Functions

79. Write as the logarithm of a single quantity:
   \( \frac{1}{2} \left[ \ln(x + 1) + 2 \ln(x - 1) \right] + \frac{1}{3} \ln x \)
80. Write as the logarithm of a single quantity:
   \( 2 \ln|3x| + \ln|x + 1| - 3 \ln|y| \)
Simplify:
81. \( e^{2\ln(x + 1)} \)
82. \( \ln(5e^3) \)
83. \( \log_3 81 \)
84. \( \log_6 108 \)
85. The spread of a flu virus through a certain population is modeled by
   \( y = \frac{1000}{1 + 990e^{-0.7t}} \)
   where \( y \) is the total number infected after \( t \) days. In how many days will 820 people be infected with the virus?
86. The amount remaining of a decaying sample is described by the function \( m(t) = m_0 2^{-t/h} \) where \( m_0 \) is the initial amount, \( t \) is the number of hours, and \( h \) is the sample's half-life. Given an isotope of sodium with a half-life of 15 hours, find the amount remaining from a 2 g. sample after 20 hours.

87. The number of bacteria in a culture is given by the formula \( n(t) = 1200e^{-0.35t} \).

What is the half-life for this type of bacteria?

**Conic Sections**

88. Find the standard equation of the parabola with vertex at \((0,0)\) and directrix \( x = 7 \).

89. Find the standard equation of the ellipse with center at \((0,0)\) a focus at \((2\sqrt{35},0)\) and minor axis of length 4.

90. Find the foci of the hyperbola:

\[ 2y^2 - 9x^2 - 18 = 0 \]

**Graphing and Translations**

91. Determine the correct function for the given graph.

(a) \(f(x) = 2x^3 - 3x^2\)  
(b) \(f(x) = 3x^4 - 2x^3\)  
(c) \(f(x) = 2x^3 + 3x^2\)  
(d) \(f(x) = 3x^2 - 2x^3\)  
(e) None of these

92. Sketch the graph of the following function: \( f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ x^2, & x > 1 \end{cases} \)

93. Graph the following function:

\[ f(x) = 4x^2 - 8x + 4 \]

94. Use the graph of \( y = x^4 \) to find a formula for the function \( y = f(x) \).

95. Given the graph of \( y = -\sqrt{x} \), graph \( y = 2 - \sqrt{x + 3} \).

96. What sequence of transformations will yield the graph of \( g(x) = (x + 1)^2 + 10 \) from the graph of \( f(x) = x^2 \)?

**Domain**

Determine the domain of the function:

97. \( f(x) = \frac{3}{x - 4} \).

98. \( f(x) = \sqrt{6 - x} \).

99. \( f(x) = \begin{cases} \sqrt{x}, & \text{if } x > 1 \\ |x|, & \text{if } x < -1 \end{cases} \)
**Asymptotes**

100. Graph the following function: \( f(x) = \frac{3 + x}{x - 1} \)

101. Graph the following rational function:
\[
f(x) = \frac{x^2}{x + 2}
\]

102. Match the graph with the correct function.

![Graph for question 102](image)

- (a) \( f(x) = \frac{x - 5}{x + 3} \)
- (b) \( f(x) = \frac{5 - x}{x + 3} \)
- (c) \( f(x) = \frac{- (x + 5)}{x + 3} \)
- (d) \( f(x) = \frac{x + 5}{x + 3} \)
- (e) None of these

**End Behavior**

Describe the end behavior of the following functions:

103. \( f(x) = \frac{x^2 - 4x - 5}{x - 3} \)

104. \( g(x) = \frac{x^3 - 2x^2 + 3}{x - 2} \)

105. \( h(x) = \frac{x^2 - 2x + 3}{x^2 + 4x - 5} \)

**Vertical and Horizontal Line Tests**

106. Use the vertical line test to determine in which case \( y \) is a function of \( x \).

- (a)
- (b)
- (c)
- (d)
- (e) None of these

107. Which of the following graphs represent \( y \) as a function of \( x \) ?

- (a)
- (b)
- (c)
- (d)
- (e) None of these

**One-to-One Functions**

Determine whether each function is one-to-one. If it is, find its inverse.

108. \( f(x) = \frac{7}{x + 2} \)

109. \( y = \sqrt[3]{x^2 + 1} \)
Inverse functions

Find the inverse of the following functions. State the domain and range of \( f^{-1}(x) \).

110. \( f(x) = -x^2 + 3 \), for \( x \geq 0 \)
111. \( f(x) = \sqrt{2x} \), for \( x \geq 0 \)
112. \( f(x) = \sqrt{25 - x^2} \), for \( 0 \leq x \leq 5 \)

Composition of Functions

113. Evaluate \( (f \circ g)(2) \) where \( f(x) = 4x \) and \( g(x) = -2x + 1 \).
114. Evaluate \( (f \circ g)(3) \) where \( f(x) = -3x + 7 \) and \( g(x) = x^2 + 4 \).
115. Evaluate \( (f \circ g)(3) \) where \( f(x) = \sqrt{x^2 + 7} \) and \( g(x) = x^2 + 1 \).

Partial Fraction Decomposition

Find the partial fraction decomposition for the following expressions:

116. \( \frac{7}{3x^2 + 5x - 2} \)
117. \( \frac{5x + 3}{x^2 - 3x - 10} \)
118. \( \frac{-5x - 3}{x^2 - 9} \)

Sequences and Series

119. Write the first five terms of the geometric sequence with \( a_1 = -3 \) and \( r = \frac{2}{3} \).
120. Find the eighth term of the arithmetic sequence with \( a_1 = 5 \) and \( d = 8 \). (Assume that \( n \) begins with 1.)
121. Determine whether the following is a geometric series, if so, find \( r \): -2, 4, -8, 16, -32…

Probability

122. The flags of seven different countries are to be displayed in a row. In how many different orders can they be flown?
123. Nine candidates are participating in an election. In how many different orders can they finish?
124. There are eight football teams in the Mountain West Conference. In how many different orders can these eight teams finish in the final rankings?
125. Six friends are driving to a basketball game. In how many different orders can they arrive?
126. A box holds 12 white, 5 red, and 6 black marbles. If 2 marbles are picked at random, without replacement, what is the probability that they will both be black?

Complex Numbers

Simplify the following complex numbers:

127. \( i^2 + 3i + 2 \)
128. \( i^3 - 2i^2 + 4i - 3 \)
129. \( i^4 + 4i^3 - 12i^2 - 13i + 1 \)

Find the conjugate of the following complex numbers:

130. \( 4 - \sqrt{-3} \)
131. \( \frac{3 + 4i}{16} \)
132. \( 3 - \sqrt{-1} \)