Math 112 – Winter 2006
Departmental Final Exam

Instructions:

• The time limit is 3 hours.

• Problem 1 consists of fill in the blank questions, each worth 1 point.

• Problem 2 consists of True or False questions, each worth 1 point.

• Problems 3 through 9 are multiple choice questions, each worth 4 points.
  Their answers MUST be entered on the grid on page 2

• Work on scratch paper will not be graded. Do not show your work for problem 1 through 9.

• Write solutions to problems 10 through 19 on the exam paper in the space provided.
  Problems 10–19 worth 6 points each. You must show your work to receive full credit.

• Please write neatly, and simplify your answers.

• Notes, books, and calculators are not allowed.

• Expressions such as \( \ln(1) \), \( e^0 \), \( \sin(\pi/2) \), etc. must be simplified for full credit.

For administrative use only:

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PART I: FILL IN THE BLANK AND TRUE OR FALSE QUESTIONS

1. (6 points) Fill in the blanks with the correct answer. Include the constant of integration where appropriate.

   (a) The derivative of $x^\pi$ equals ________________________________

   (b) The derivative of $\frac{x}{1 + x}$ equals ________________________________

   (c) The derivative of $\ln(1 + x^2)$ equals ________________________________

   (d) The integral $\int \sin 2x \, dx$ equals ________________________________

   (e) The integral $\int_0^1 2xe^{x^2} \, dx$ equals ________________________________

   (f) The derivative of $\tan^2 x \, dx$ equals ________________________________

2. (6 points) Mark T if true under all circumstances; mark F otherwise.

   (a) ______ $\lim_{x \to 0} \frac{\sin 4x}{3x} = \frac{4}{3}$

   (b) ______ If $y = f^{-1}(x)$ denotes the inverse of $f(x)$, then by chain rule $\frac{dy}{dx} = -\frac{f'(x)}{f^2(x)}$.

   (c) ______ If $y = 10^x$ then $\frac{dy}{dx} = x \, 10^{x-1}$.

   (d) ______ $\frac{d}{dx} \int_0^{2x} f(t) \, dt = 2f(2x)$.

   (e) ______ If $f(x)$ has a removable discontinuity at $x = 0$, then $f(x)$ can be defined at $x = 0$ so that $f$ is differentiable at $x = 0$.

   (f) ______ If $f''(c)$ exists and $(c, f(c))$ is an inflection point for $y = f(x)$, then $f''(c) = 0$. 
3. Assume that $f$ is a continuous function on $[a, b]$. The partition of $[a, b]$ is given as $P = \{a = x_0 < x_1 < \cdots < x_{i-1} < x_i < \cdots < x_n = b\}$. Let $\Delta x_i = x_i - x_{i-1}$, $x_{i-1} \leq t_i \leq x_i$, and $\|P\| = \max\{\Delta x_1, \Delta x_2, \cdots, \Delta x_n\}$. Which of the following is not equal to $\int_a^b f(x)dx$?

(a) $\lim_{\|P\| \to 0} \sum_{i=1}^n f(t_i) \Delta x_i$
(b) $\lim_{\|P\| \to 0} \sum_{i=1}^n f(x_i) \Delta x_i$
(c) $\lim_{\|P\| \to 0} \sum_{i=1}^n f(x_{i-1}) \Delta x_i$
(d) $\lim_{\|P\| \to 0} \sum_{i=1}^n f\left(\frac{x_i - x_{i-1}}{2}\right) \Delta x_i$
(e) $\lim_{\|P\| \to 0} \sum_{i=1}^n f\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x_i$. 

4. Consider $\lim_{x \to 12} \left(\frac{x}{4} - 2\right) = 1$. If $\epsilon > 0$ is given, find the largest possible $\delta$ such that $0 < |x - 12| < \delta$ implies that $\left|\left(\frac{x}{4} - 2\right) - 1\right| < \epsilon$.

(a) $\delta = 12\epsilon$  (b) $\delta = 4\epsilon$  (c) $\delta = 2\epsilon$  (d) $\delta = \epsilon$  (e) $\delta = 6\epsilon$

(f) $\delta = 3\epsilon$  (g) $\delta = 5\epsilon$
5. Find the average value of \( f(x) = 4x - x^2 \) on \([0, 4]\)

(a) \( \frac{2}{3} \)  
(b) \( \frac{4}{3} \)  
(c) \( \frac{8}{3} \)  
(d) \( \frac{16}{3} \)  
(e) \( \frac{32}{3} \)  
(f) 0  
(g) 32  
(h) 16  
(i) 8  
(j) None of these.

6. Find \( f^{(13)}(\pi/4) \), the 13\textsuperscript{th} derivative of \( f \) at \( x = \pi/4 \), for \( f(x) = \sqrt{2} \cos x \).

(a) \(-4\)  
(b) 4  
(c) \(-2\)  
(d) 2  
(e) \(-1\)  
(f) 1  
(g) 0

7. If \( F'(x) = f(x) \), then \( \int_a^b f(x) dx = \)

(a) \( f(a) - f(b) \)  
(b) \( f(b) - f(a) \)  
(c) \( f'(b) - f'(a) \)  
(d) \( F'(b) - F'(a) \)  
(e) \( F(a) - F(b) \)  
(f) \( F(b) - F(a) \)  
(g) \( f(b) \)  
(h) None of these.

8. Find \( \frac{dy}{dx} \) at \( x = \pi/6 \) if \( y = 2 \tan^{-1}(2 \sin x) \).

(a) \( 2\sqrt{3} \)  
(b) 2  
(c) \( \pi/2 \)  
(d) \( \sqrt{3} \)  
(e) \( 4\sqrt{3} \)  
(f) \( \pi/3 \)  
(g) \( 1/\sqrt{3} \)
9. The function $f(x) = x^3 - x^6$
   (a) increases on $(-\infty, 0)$, and decreases on $(0, \infty)$.
   (b) $f^{(6)} = 0$.
   (c) $x = 1$ is a critical point.
   (d) is concave down on $(-\infty, 0)$, and concave up on $(0, \infty)$.
   (e) is concave up on $(0, 5^{-1/3})$ and concave down on $(5^{-1/3}, \infty)$, and $x = 5^{-1/3}$ is an inflection point.
   (f) has local maximum at $x = 0$, and $x = 0$ is an inflection point.
   (g) has local minimum at $x = 2^{-1/3}$.

The answers to the multiple choice MUST be entered on the grid on page 2. Otherwise, you will not receive credit.
Part III: Written Solutions

For problems 10 - 19, write your answers in the space provided. Neatly show your work for full credit.

10. The equation \( x \ln y = y^3 \) implicitly defines \( y \) as a function of \( x \). Find \( \frac{dy}{dx} \).

11. (a) Evaluate the definite integral \( \int_{-2}^{2} \sqrt{4 - x^2} \, dx \) as an area.

(b) Use linearity properties and the properties of even and odd functions to evaluate the definite integral \( \int_{-1}^{1} (3x^4 \tan x - x^2) \, dx \).
12. Suppose the radius of a spherical object is expanding at a rate of 0.5 m/hr. How fast is the volume increasing when the radius is 5m?

13. Find the limit. If the limit does not exist, write “Limit = ∞,” or “Limit = −∞,” or if neither of these is true, write “Limit doesn’t exist.” You must show your work.

(a) \( \lim_{x \to \infty} \left( x - \sqrt{x^2 - 5x} \right) \)

(b) \( \lim_{x \to 0^+} \left( 1 + \frac{3}{x} \right)^x \)
14. A hot ingot is removed from the furnace and placed for cooling outdoors, where the temperature is 70°F. After one hour, the temperature of the ingot is 250°, and after 2 hours, the temperature is 100°. How hot was the ingot when it was removed from the furnace?

(Newton’s cooling model is \( \frac{dT}{dt} = -k(T - A) \).)

15. Find \( \frac{df}{dx}(0) \) for

(a) \( f(x) = \sec(3x) \tan(4x) \).

(b) \( f(x) = u^3 + 1, \ u = 3v - 2, \ v = x^3 + 1 \).
16. Compute the following integrals:

(a) \( \int \frac{6}{\sqrt{x}(1 + \sqrt{x})} \, dx \)

(b) \( \int \frac{\sin(\ln x)}{x} \, dx \)

17. (a) Find the horizontal asymptote of \( f(x) = \frac{x + 1}{x - 1} \), and sketch the graph of \( f \).

(b) Show that the polynomial \( p(x) = x^3 - x^2 - 2x + 2 \) has a zero in the interval \([-2, 0]\).
18. A piece of commercial property is to be enclosed by fencing on the front and two sides. Fencing for the sides costs $2.50 per foot, and fencing for the front costs $4.00 per foot. What are the dimensions of the largest rectangular lot that can be so enclosed for $800?

19. a) State the definition of derivative of $f(x)$ at the point $a$.

b) Use definition in a) to find the derivative of $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & x \neq 0; \\ 0, & x = 0 \end{cases}$ at $x = 0$. 