1. Prove that if $R$ is a commutative ring with 1, then every maximal ideal of $R$ is prime.

2. Let $p$ and $q$ be distinct primes. Prove that any group of order $pq$ is solvable.

3. Define the commutator subgroup $G'$ of a group $G$, and prove that if $N$ is a normal subgroup of $G$ such that $G/N$ is abelian, then $G'$ is a subgroup of $N$.

4. Determine (with proof) a complete set of representatives of the conjugacy classes of the group $\text{GL}_3(\mathbb{F}_2)$. Be sure that your list has no repetition.

5. Let $m$ and $n$ be positive integers. Compute (with justification) $(\mathbb{Z}/m\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/n\mathbb{Z})$.

6. Let $R$ be a commutative ring with 1. Prove carefully that every proper ideal of $R$ is contained in a maximal ideal of $R$.

7. Let $R$ be a commutative ring with 1, and let $I_1, \ldots, I_n$ be ideals of $R$. If
   \[ J = I_1 \cap I_2 \cap \cdots \cap I_n \]
   is a prime ideal of $R$, show that at least one of the ideals $I_k$, with $k \in \{1, \ldots, n\}$, is prime.

8. Find the Galois group of $x^5 - 4x^3 + 2$ over $\mathbb{Q}$.

9. Prove that no finite field is algebraically closed.

10. Determine (with proof) the number of irreducible degree 6 polynomials in $\mathbb{F}_2[x]$. 