1. Do continuous functions preserve sets of measure zero? Prove or disprove.

2. $f$ is (extended) real-valued. Are these two statements equivalent?

   - $f^{-1}(E)$ is measurable for every interval $E$.
   - $f^{-1}(E)$ is measurable for every Borel set $E$.

   Prove your answer.

3. Are the real and imaginary parts of a holomorphic function harmonic? Prove.

4. (a) What is the precise statement of the open mapping theorem for linear mappings in Banach spaces?

   (b) What is the precise statement of the Hahn-Banach Theorem?

   (c) What is the precise statement of the Riesz Representation Theorem for Hilbert Spaces?

   (d) What is a precise statement of the Lebesgue-Radon-Nikodym Theorem?

5. For sequences of complex-valued measurable functions, what relationships exist between convergence in measure and point-wise convergence? Give proofs and examples.

6. Suppose $\{f_n\}$ is a sequence of holomorphic functions in some non-empty connected open set $\Omega$. Suppose this sequence converges uniformly on compact subsets of $\Omega$ to a function $f$. Discuss $f$.

7. Prove: If $f, g \in L^1(\mathbb{R}^n)$, $f \ast g$ is convolution and $\hat{f}$ represents Fourier transform of $f$ then $h = f \ast g$ is in $L^1$ and $\hat{h}(t) = \hat{f}(t)\hat{g}(t)$. 

8. Let $H$ be a normed linear space. Must $H^*$ (the dual of $H$) be a Banach space? Prove your answer.

9. Let $\gamma$ be the positively oriented unit circle. Compute

$$\frac{1}{2\pi i} \int_{\gamma} \frac{e^z - e^{-z}}{z^4} dz.$$

10. Suppose $\mu$ is a positive measure on $X$, $0 < p < \infty$, $f \in L^p(\mu)$, $f_n \to f$ a.e., and $\|f_n\|_p \to \|f\|_p$ as $n \to \infty$. Prove that

$$\lim_{n \to \infty} \|f_n - f\|_p = 0.$$