** The final exam will be available in the JSB Auditorium anytime that it is open during finals week. It is not comprehensive (but you will still need to know basic concepts from the other sections in order to solve these problems, such as the unit circle) and covers sections 4.1-4.5 and 5.1-5.3.

** All homework is due by Wednesday, April 13th, at 11:59pm (except for the last assignment, which is due the following day, Thursday, April 14th.

** Check your grade on the gradebook.byu.edu to make sure that all test scores, extra credit, and changed scores have been entered correctly. E-mail me by Friday (April 22) evening about anything that needs changed.

** SAMPLE QUESTIONS: - know how to do these questions and questions similar to these (as well as main concepts from the HW and class discussions)

Assume for the following questions that side \(a\) opposite of angle \(A\), side \(b\) is opposite of angle \(B\), and side \(c\) is opposite of angle \(C\) whenever triangles are discussed (Hint: draw the triangles)

1) Given a right triangle where \(C=90^\circ\), \(B=23^\circ\), and \(b=5\), find the length of side \(a\).
   ** Also be able to solve these types of problems within a story context such as angle of elevation of the sun and shadows.

2) Using identities and the complementary angle theorem, simplify the following expression:
   \[1 - \cos^2 20^\circ - \cos^2 70^\circ\]

3) Determine whether the given information results in one triangle, two triangles, or no triangle at all. \(B=40^\circ\), \(b=4\), and \(c=6\).

4) Given that \(A=15^\circ\), \(B=35^\circ\), and \(c=7\), find the value of \(a\).

5) Given that \(a=4\), \(b=1\), and \(C=120^\circ\), find the value of \(c\).

6) Given that \(a=9\), \(b=7\), and \(c=10\), find the value of \(B\).
   ** Again, also be able to solve these types of problems within a story context.

7) Given that \(b=3\), \(c=2\), and \(A=110^\circ\), find the area of this triangle.

8) Given that \(a=4\), \(b=3\), and \(c=6\), find the area of this triangle.

9) A giant pendulum is being swung back and forth. It takes 10 seconds for one full swing (front to back to front). When the pendulum is swung out the farthest, it makes an angle of 45° with the vertical. If the pendulum is 50 feet long and we ignore all resistive forces, write an equation that relates the horizontal displacement of the pendulum from its rest position after time \(t\). [Hint: the horizontal displacement would follow simple harmonic motion].

10) An object with a mass of 20 grams is attached to the end of a coil spring and is pulled down a distance of 15 cm. from its rest position and then released. There is a damping factor of .75 grams/second. Assume that the positive direction of motion is up and the object takes 6 seconds to go one full oscillation (under simple harmonic motion). Write an equation that relates the displacement \(d\) (in centimeters) of the object from its rest position after \(t\) seconds.
11) Sketch the graph of the damped vibration curve \( d(t) = e^{-t/\pi} \cos (3t) \).

12) Find multiple ways to describe the point \( \left( 3, \frac{4\pi}{3} \right) \) given in polar coordinates. Also give the rectangular coordinates.

13) Convert the point \( \left( -4, \frac{4\sqrt{3}}{3} \right) \) given in rectangular coordinates to polar coordinates (multiple answers).

14) Convert the following equation using rectangular coordinates to an equation using polar coordinates: \( 5x^2 + 5y^2 = 2y \)

** Be able to convert the other way as well

15) Graph the following polar equation: \( r = 3 - 3 \sin \theta \)

16) Graph the following polar equation: \( r = 3 \sin (5\theta) \)

17) If \( z = 10(\cos 68^\circ + i \sin 68^\circ) \) and \( w = 5(\cos 52^\circ + i \sin 52^\circ) \) are complex numbers, find the product \( z \cdot w \) (leave your answer in standard polar form \( a+bi \)).

18) Using De Moivre’s Theorem, what is the complex number \( [4(\cos 15^\circ + i \sin 15^\circ)]^3 \) written in the standard rectangular form \( a + bi \)?
ANSWERS:
1) \( a = 5 \cot(23^\circ) \)
2) 0
3) two triangles
4) \( a = 7 \cdot \sin(15^\circ) / \sin(130^\circ) \)
5) \( \sqrt{21} \)
6) \( \cos^{-1} \frac{11}{15} \)
7) \( 3 \sin 110^\circ \)
8) \( \frac{\sqrt{455}}{4} \)
9) \( d(t) = 25\sqrt{2} \cos \left( \frac{\pi}{5} t \right) \)
10) \( d(t) = -15e^{-\frac{75t}{40}} \cos \left( \sqrt{\frac{\pi^2}{9} - \frac{75^2}{1600}} t \right) \)

11) [Graph of a cosine function]

12) Some examples include \((3, \frac{10\pi}{3}), (3, \frac{-2\pi}{3}), (-3, \frac{\pi}{2}), (-3, \frac{-5\pi}{3})\), etc. The rectangular coordinates would be \((-\frac{3}{2}, -\frac{3\sqrt{3}}{2})\).

13) Some possible answers are \((\frac{8\sqrt{3}}{3}, \frac{5\pi}{6}), (-\frac{9\sqrt{3}}{3}, -\frac{\pi}{6})\), etc.

14) \(5r^2 = 2r \sin \theta\)

15) [Graph of a circle]

16) [Graph of a star]

17) \( z \cdot w = 50(\cos 120^\circ + i \sin 120^\circ) = 50 \left( -\frac{1}{2} + i \left( \frac{\sqrt{3}}{2} \right) \right) \rightarrow z \cdot w = -25 + 25\sqrt{3}i \)

18) \(32\sqrt{2} + 32\sqrt{2} i\)
Formulas needed (also certain main formulas from the other chapters may apply):

*Again, assume these equations for a triangle with sides $a$, $b$, $c$ and opposite angles $A$, $B$, $C$, respectively.

Complementary Angle Theorem: Cofunctions of complementary angles are equal

$$A + B + C = 180^\circ$$

Law of Sines: \[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \] (Know when there would be no triangle, 1 triangle, or 2 triangles also)

Law of Cosines: (keep in mind that these are all the same formula)

\[ \begin{align*}
c^2 &= a^2 + b^2 - 2ab \cos C \\
b^2 &= a^2 + c^2 - 2ac \cos B \\
a^2 &= b^2 + c^2 - 2bc \cos A
\end{align*} \]

Area of a Triangle: The area $K = \frac{1}{2} ab \sin C$

Heron's Formula: The area of a triangle is $K = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$.

Simple Harmonic Motion: An object that moves on a coordinate axis so that the distance $d$ from its rest position at time $t$ is given by either $d = a \cos(\omega t)$ or $d = a \sin(\omega t)$ where $a$ and $\omega > 0$ are constants, moves with simple harmonic motion. The motion has amplitude $|a|$ and period $\frac{2\pi}{\omega}$.

Damped Motion: The displacement $d$ of an oscillating object from its at-rest position at time $t$ is given by

$$d(t) = a e^{-bt/(2m)} \cos \left( \sqrt{\omega^2 - \frac{b^2}{4m^2}} \; t \right)$$

where $b$ is the damping factor and $m$ is the mass of the oscillating object. Here $|a|$ is the displacement at $t=0$ and $\frac{2\pi}{\omega}$ is the period under simple harmonic motion (no damping).

** Know the equations for Circles, Cardioids, Limaçons, Roses, and Lemniscates in polar equations (or whatever you need in order to graph them) – pg 317, 319-320 in the book

Complex Numbers: $z = x + yi$ in polar form is $z = r(\cos \theta + i \sin \theta)$ with $r \geq 0$ and $0 \leq \theta < 2\pi$

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be two complex numbers. Then

$$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)] \quad (5)$$

If $z_2 \neq 0$, then

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)] \quad (6)$$

De Moivre's Theorem

If $z = r(\cos \theta + i \sin \theta)$ is a complex number, then

$$z^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

where $n \geq 1$ is a positive integer.