315 Objectives, Chapter 4

Section 4.1 Discussion: Examples of Dirichlet and Thomae
1. Be able to write the definition of Dirichlet’s function and Thomae’s function.
2. Know the continuity and discontinuity properties of Thomae’s function.

Section 4.2 Functional Limits
1. Know and understand the definition (4.2.1) of limit of a function.
2. Know theorem 4.2.3 (connection of sequential limits and functional limits) and be able to use it in proofs such as exercise 5a and example 4.2.6 (p. 107).
3. Be able to write an $\epsilon, \delta$ proof of a specific functional limit (such as $\lim_{x \to 2} x^2 = 4$).

Section 4.3 Combinations of Continuous Functions
1. Know the definition of continuity (Def. 4.3.1).
2. Know the sequential characterization of continuity (Thm. 4.3.2 (iv))
3. Be able to use the (sequential) criterion for discontinuity (Corollary 4.3.3) to prove a function is discontinuous at a given point. oscillating discontinuity, as in example 4.2.6).
4. Know and be able to prove theorem 4.3.4, the Algebraic Continuity Theorem.
5. Be able to prove theorem 4.3.9 (composition of continuous functions is continuous)

Section 4.4
1. (T) Be able to prove theorem 4.4.2. (A continuous functions maps a compact set to a compact set)
2. (T) Be able to prove theorem 4.4.3. (The extreme value theorem)
3. (T) Know the definition of uniform continuity. (Defn. 4.4.5)

4. (T) Be able to recognize whether or not a given function \( f : D \to \mathbb{R} \) is continuous on \( D \), and also whether or not \( f \) is uniformly continuous on \( D \).
   Helpful examples include:
   \[
   f(x) = x^2 \text{ on } [0, 100] \\
   f(x) = x^2 \text{ on } \mathbb{R} \\
   f(x) = 3x \text{ on } \mathbb{R} \\
   f(x) = |x|/x \text{ on } [-1, 0) \cup (0, 1] \\
   f(x) = \sqrt{x} \text{ on } [0, \infty) \\
   f(x) \text{ continuous on a compact domain.}
   \]

5. (T) Be able to use theorem 4.4.6 (sequential criterion for nonuniform continuity) to prove that a given function is not uniformly continuous.

6. (T) Be able to solve problems like the assigned homework problems 1bc and 2.

**Section 4.5**

1. (T) Be able to state the intermediate value theorem. This can be tricky so you should practice. :-)

2. Be able to prove the intermediate value theorem using the method of bisection (which is based on the Nested Interval Property).