Objectives for Math 315
315 Objectives, Chapter 5

Section 5.1 Discussion: Are derivatives continuous?
1. (T) Be able to prove that
   \[ f(x) = \begin{cases} 
   x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\
   0 & \text{if } x = 0 
   \end{cases} \]
is differentiable everywhere, but that its derivative is discontinuous at 0.

Section 5.2 Derivatives and the intermediate value property
1. Know the definition of derivative.
2. Be able to work out (from the definition) the derivative of \( f(x) = x^n \) for any positive integer \( n \).
3. Know Theorem 5.2.3: If \( f \) is differentiable then \( f \) (though maybe not \( f' \)) is continuous.
4. Be able to prove Theorem 5.2.4 parts (i) to (iii), the sum and product rules for differentiation.
5. Know the chain rule.
6. Be able to prove the interior extremum theorem, Thm. 5.2.6.
7. Know Darboux’s theorem.

Section 5.3: The mean value theorem
Underlying ideas:
1. If you know \( f'(x) \) (or bounds on it) at every point of an interval, and you want information about the function \( f(x) \) itself, use the mean value theorem.
2. Examples: The fundamental theorem of calculus, proving minimization, L’Hospital’s rule.
3. In the first two cases, you invent a function (described in \textbf{words}, not as a formula), then figure out a \textbf{formula} for its derivative (or bounds on the derivative), then translate that information to a formula (or bound) for the function itself.

Skills
1. Be able to state and prove the mean value theorem.
2. Be able to prove corollaries 5.3.3 and 5.3.4.
3. Know the definition of \( \lim_{x\to c} f(x) = \infty \).
4. Cancelled (no objectives on L’Hospital’s rule).

Sections 5.4, 5.5: A continuous, nowhere differentiable function, and Epilogue
1. Be able to define the continuous, nowhere differentiable function \( g(x) \) from Exercise 5.4.1.