1. If the columns of an orthogonal matrix are permuted, prove that the result is still an orthogonal matrix.

2. Are the matrices \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 3
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
1 & 4 & 5 \\
0 & 2 & 6 \\
0 & 0 & 3
\end{bmatrix}
\] similar? Explain.

3. Let \( U \) be a finite-dimensional vector space, and let \( V \) and \( W \) be subspaces of \( U \). Give a formula relating the dimensions of \( V \), \( W \), \( V + W \), and \( V \cap W \). Prove that your formula is correct. (Note: You may use without proof the vector space analogues of the homomorphism theorems from group theory.)

4. Show that there is no simple group of order 148.

5. Let \( H \) be a subgroup of finite index of an infinite group \( G \). Prove that \( G \) has a normal subgroup \( K \) of finite index in \( G \) with \( K \subset H \).

6. Determine the last 3 digits of the number \( 13^{2011} \). Explain your method.

7. (a) Let \( A \) be a commutative ring with 1. An element \( a \in A \) is said to be nilpotent if \( a^n = 0 \) for some positive integer \( n \). Prove that the nilpotent elements of \( A \) form an ideal in \( A \).

(b) Does the result of part (a) still hold if the hypothesis of commutativity is dropped? Prove or disprove.

8. Let \( R \) be a commutative ring with 1. Prove that the principal ideal \((x)\) in the polynomial ring \( R[x] \) is a maximal ideal if and only if \( R \) is a field.

9. Prove that the Galois group of the splitting field of \( x^4 - 2 \) over \( \mathbb{Q} \) has order 8 and contains an element of order 4.

10. Let \( F \) be a field with 81 elements. Does the polynomial \( x^2 + 1 \) have a root in this field? (The polynomial should be considered as having coefficients in \( \mathbb{Z}/3\mathbb{Z} \).)