1. When we say that \((\mathbb{R}, +, \cdot, \leq)\) is a complete ordered field, we are asserting more than that \((\mathbb{R}, +, \cdot)\) is a field and \((\mathbb{R}, \leq)\) is a totally ordered set. What else are we asserting? (Be specific.)

2. Prove that if \(\{a_n\}_{n=1}^{\infty}\) is a sequence of real numbers converging to \(A\) and \(\{b_n\}_{n=1}^{\infty}\) is a sequence of nonzero real numbers converging to \(B \neq 0\), then \(\{a_n/b_n\}_{n=1}^{\infty}\) converges to \(A/B\).

3. Let \(f : X \to Y\) be a continuous function between metric spaces. If \(X\) is compact, prove that \(f\) is uniformly continuous.

4. Let \(f : \mathbb{R}^n \to \mathbb{R}^n\) be a continuous function satisfying \(\|f(x)\| < \|x\|\) for every \(x \neq 0\). For some initial point \(x_0 \in \mathbb{R}^n\) define the sequence \(\{x_k\}_{k=0}^{\infty}\) recursively by the rule \(x_{k+1} = f(x_k)\). Show that the sequence converges to \(0\).

5. Let \(f : \mathbb{R} \to \mathbb{R}\) be a uniformly continuous function on \(\mathbb{R}\). Prove that there exists positive constants \(a\) and \(b\) such that 
   \[|f(x)| \leq a|x| + b, \quad \forall x \in \mathbb{R}.\]

6. Let \(U \subset \mathbb{R}^2\) be a nonempty open set. Prove that there is no one-to-one continuously-differentiable function mapping \(U\) into \(\mathbb{R}\).

7. Let \(f : [0, 1] \to \mathbb{R}\) be defined by the formula
   \[f(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Q} \\ 1/q & \text{if } x \in \mathbb{Q} \text{ and } x = p/q \text{ in lowest terms}. \end{cases}\]

   Determine with proof the value of the (Riemann-Darboux) integral \(\int_0^1 f(x) \, dx\) or that the integral is undefined.

8. Evaluate \(\int_0^1 \int_0^1 \int_0^1 \sqrt{x^3 + z} \, dz \, dx \, dy\).

9. For \(a > b > 0\), evaluate the integral \(\int_{-\infty}^{\infty} \frac{\cos x \, dx}{(x^2 + a^2)(x^2 + b^2)}\).

10. Suppose that \(f\) is analytic in \(|z| < 1\) and that \(|f(z)| < \frac{1}{1 - |z|}\) for \(|z| < 1\). Show that for \(0 < R < 1\),
    \[|f^{(n)}(0)| \leq \frac{1}{R^n(1 - R)} \quad (n = 1, 2, \ldots).\]

    What choice of \(R\) yields the best upper bound?