Applied Math Syllabus for PhD Qualify Exam  
Math 634 and Math 635

1. Existence and Uniqueness of Solutions to Ordinary Differential Equations:  
   Existence of solutions;  
   The contraction mapping principle.  
   Uniqueness of solutions;  
   Gronwall inequality.  
   Continuation of solutions;  
   Dependence of solutions on initial conditions and Parameters;

2. Linear Differential Equations and Linearization:  
   Linear systems with constant coefficients;  
   Jordan normal form of matrices, fundamental solutions.  
   Nonautonomous linear systems;  
   Fundamental metric solutions, the variation of constants formula.  
   Floquet theory for periodic systems

3. Stability of Equilibrium Points  
   Stability and asymptotic stability  
   Lyapunov functions  
   Gradient Systems  
   Phase-plane analysis

4. Poincare-Bendixon Theory  
   Invariant sets, omega-limit sets, alpha-limit sets  
   Limit cycles  
   Van der Pol’s equation

5. Stability of Nonautonomous Equations  
   Stability, asymptotic stability, uniform stability, uniformly asymptotic stability  
   Stability of linear systems  
   Existence of Periodic orbits  
   Fredholm Alternative  
   Stability of Periodic orbits  
   Orbital stability  
   Poincare section and Poincare maps

6. Stable and Unstable Manifolds

7. Discrete Dynamical Systems  
   Stability of Critical Points  
   Stable and unstable manifolds  
   Structural stability, Hartman-Grobman theorem  
   Smooth conjugacy  
   Bifurcations  
   Dynamical bifurcations, bifurcations from simple Eigenvalues, global bifurcations, Crandall-Robinowitz Theorem, Periodic doubling bifurcations, Feigenbaum universal numbers  
   Symbolic dynamics and Smale Horseshoe maps, transversal homoclinic orbits

8. Center Manifolds  
   Hopf Bifurcations

9. Normal form theory

10. Averaging and local bifurcations

11. Homoclinic Bifurcations, Melnikov functions  
    Exponential dichotomy, shadowing lemma  
    Uniform Contraction Principle. Gronwall’s Inequality. Variation of Constants Formula

    Stability of Equilibrium, Lyapunov Fuctions, Gradient Systems, Invariant Set, Omega-limit set,  
    Alpha-limit set, Poincare-Bendixon Theorem, Floquet Theory, Existence and Stability of  
    Periodic Solutions, Stable and Unstable Manifold Theorem, Center Manifold Theorem,  
    Hartman-Grobman Theorem,