1. Let $G$ be a group of order 9. Prove that $G$ is Abelian.

2. Let $a$ and $b$ be elements of a group. Prove that the elements $ab$ and $ba$ have the same order.

3. Let $\alpha$ and $\beta$ be transpositions (2-cycles) in $S_n$, with $\alpha \neq \beta$. Let $\gamma = \alpha \beta$ and prove that $\gamma$ is either a 3-cycle, or a product of two 3-cycles.

4. Let $S = \left\{ \frac{m}{2n+1} : m, n \in \mathbb{Z}, \gcd(m, 2n+1) = 1 \right\}$. Let $I = \left\{ \frac{m}{k} \in S : m \text{ is even } \right\}$. Prove that $S/I \cong \mathbb{Z}_2$.

5. Let $R$ be a commutative ring with identity, and suppose that $R$ has no ideals except 0 and $R$. Prove that $R$ is a field. Explain why the theorem is false if $R$ is not commutative.

6. Let $A$ be an $m \times n$ matrix with real entries. Show $\text{Row } A = (\text{Nul } A)^\perp$.

7. Let $A$ be a real $k \times k$ matrix with $k$ distinct eigenvalues; where each eigenvalue is in the interval $(0,1)$. Prove that $A^n \to (0)$ (the matrix of zeros) as $n \to \infty$.

8. A submatrix of $A$ is any matrix that results from deleting some (or none) of the rows and/or columns of $A$. Prove that if the rank of $A$ is $r$ then $A$ contains an invertible $r \times r$ submatrix.

9. Let $K$ and $F$ be fields. If $K \supset F$ is such that $[K : F] = p$, $p$ a prime, show that $K = F(a)$ for every $a$ in $K$ that is not in $F$.

Now answer only one of:

10. Prove that the Galois group of $x^5 - 6x + 3$ over the rationals is $S_5$, so this polynomial cannot be solved by radicals.

11. Let $G = D_8 = \langle a, b : a^4 = b^2 = 1, b^{-1}ab = a^{-1} \rangle$ (the dihedral group generated by elements $a$ and $b$). Let $H$ be the subgroup $\langle a^2, b \rangle$. Define $U$ to be the 1-dimensional subspace of $\mathbb{C}H$ spanned by

$$1 - a^2 + b - a^2b.$$ Show that $U$ is a $\mathbb{C}H-$submodule of $\mathbb{C}H$. 