1. A matrix $A$ is said to be idempotent if $A^2 = A$. Show that every finite, real, symmetric idempotent matrix represents a linear transformation which is an orthogonal projection and identify the subspace onto which it is projecting.

2. Prove that any set of 3 vectors in $\mathbb{R}^2$ is linearly dependent.

3. State and prove the Rank Plus Nullity theorem for $\mathbb{R}^n$.

4. Let $G$ be a group. Show that if each element of $G$ has order 2, then $G$ is abelian.

5. If $G$ is a group, show that if $H$ is a normal subgroup of $G$, then the multiplication of right cosets $Hg_1Hg_2 = Hg_1g_2$ is well defined.

   Find an example of a group $G$ with a subgroup $H$ for which this multiplication would not be well defined.

6. Let $G$ be a group of order 245. Show that $G$ has a normal subgroup of order 49.

7. Find an ideal in $\mathbb{Z} \times \mathbb{Z}$ that is prime but not maximal.

8. Let $R$ be a commutative ring with 1 $\neq 0$. Suppose $I$ and $J$ are ideals of $R$ such that $I + J = R$. Prove that

   $$R/(I \cap J) \cong R/I \oplus R/J.$$ 

9. Prove that the Frobenius map on a field of finite characteristic is always injective and give an example of a field of finite characteristic for which it is not surjective.

10. Prove that the order of a finite field is a power of a prime.