1. Let $V$ be the vector space of polynomials in $x$ over $\mathbb{C}$ of degree less than 5. Let $T : V \to V$ be $T(f) = f'' - 6f' + 9f$. Show that $T$ is a linear transformation and find its kernel.

2. Prove that if the eigenvalues of a diagonalizable matrix are all $\pm 1$, then the matrix is its own inverse.

3. Find a symmetric matrix similar to $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ or prove that such a matrix does not exist.

4. Show that every index 2 subgroup of a group is normal. Give an example (with proof) of a group with a non-normal index 3 subgroup.

5. Given an action of $G$ on a set, show that every point of some orbit has the same stabilizer if and only if this stabilizer is a normal subgroup of $G$.

6. Prove that if $G$ is a cyclic group and $N$ is a normal subgroup of $G$, then $G/N$ is a cyclic group.

7. Let $R$ be a commutative ring with 1 and with elements $a, b$ of additive orders 5 and 7 respectively. Is $R$ an integral domain?

8. If $I$ is an ideal of the ring $R$, prove that $I[x]$ is an ideal in the polynomial ring $R[x]$ and that $R[x]/I[x] \cong (R/I)[x]$.

9. Let $K = \mathbb{Q}(\zeta)$, where $\zeta$ is a primitive 5th root of unity.
   
   (a) Prove that $\text{Gal}(K/\mathbb{Q})$ is cyclic of order 4.
   
   (b) Find (with proof) a polynomial $g(x) \in \mathbb{Q}[x]$ such that the unique quadratic subfield of $K/\mathbb{Q}$ is $\mathbb{Q}(\alpha)$, where $\alpha$ is a root of $g(x)$.

10. Determine the minimal polynomial over $\mathbb{Q}$ of the element $\beta = \sqrt{2} + \sqrt{5}$. 