1. Find a $3 \times 3$ orthogonal matrix with all entries nonzero. (No partial credit.)

2. Find the dimension of the space of $n \times n$ matrices with trace 0. Justify your answer. (The trace of a square matrix is the sum of its diagonal entries.)

3. Let $V$ and $W$ be finite-dimensional vector spaces, and let $L : V \to W$ be a linear transformation. Prove that $\dim(\ker(L)) + \dim(\text{image}(L)) = \dim(V)$.

4. Let $p, q$ be primes. Show that a group of order $pq$ is not simple.

5. Let $Q$ be the quaternion group of order 8. Prove that every subgroup of $Q$ is normal.

6. Let $G$ be a group with center $Z(G)$. Prove that if $G/Z(G)$ is cyclic, then $G$ is abelian.

7. Let $R$ be a commutative ring with 1. Let $x, y \in R$ and let $I$ be the smallest ideal containing $x$ and $y$. Show that $I = \{ax + by : a, b \in R\}$.

8. In a principal ideal domain, show that every nonzero prime ideal is maximal.

9. Calculate the 36th cyclotomic polynomial $\Phi_{36}(x)$.

10. Let $F$ be a field with 125 elements. Does the polynomial $x^2 + x + 1$ have a root in $F$? (The polynomial should be considered as having coefficients in $\mathbb{Z}/5\mathbb{Z}$.) Justify your answer.