

Complete the following exam and record your answers on the bubble sheet provided.

1. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
 - (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (f) None of the above.

2. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
 - (a) 700 steel, 900 lumber
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - (f) None of the above.

3. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.
 - (a) 6600
 - (b) 2400
 - (c) 5400
 - (d) 900
 - (e) 3300
 - (f) None of the above.

4. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
- (a) 0
 - (b) 3
 - (c) -6
 - (d) -9
 - (e) There is no solution to this problem.
 - (f) None of the above.
5. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.
6. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) (1, 10)
 - (b) (0, 12)
 - (c) (0, 0)
 - (d) (0, 11)
 - (e) (6, 0)
 - (f) None of the above.
7. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
- (b) -1
- (c) 4
- (d) 1
- (e) 6
- (f) None of the above.

8. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.

9. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ \hline -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
 - (b) $(0, 3, 2)$
 - (c) $(12, 20, 60)$
 - (d) $(10, 0, 0)$
 - (e) This is not a valid tableau.
 - (f) None of the above.
10. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
- (a) BC
 - (b) $A + C$
 - (c) CA
 - (d) AB
 - (e) BB
 - (f) None of the above.

11. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

- (a) 1
- (b) -1
- (c) -2
- (d) -3
- (e) 3
- (f) None of the above.

12. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.

- (a) 4
- (b) 1
- (c) 5
- (d) 3
- (e) 2
- (f) None of the above.

13. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ \hline 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
 - (b) 76
 - (c) 12
 - (d) 6
 - (e) The objective does not increase.
 - (f) None of the above.
14. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ \hline -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) (2, 1)
- (b) (2, 3)
- (c) (3, 1)
- (d) (3, 2)
- (e) Pivoting is not the correct action for this tableau.
- (f) None of the above.

15. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.

(a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$

(b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$

(c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$

(d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$

(e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$

(f) None of the above.

16. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

(a) -8

(b) 10

(c) 1

(d) -1

(e) 8

(f) None of the above.

17. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
- (a) $30x + 25y$
 - (b) $x + 0.5y$
 - (c) $2x + 5y$
 - (d) $2x + 25y$
 - (e) $30x + 0.5y$
 - (f) None of the above.
18. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.
- (a) 0
 - (b) -17
 - (c) -5
 - (d) -1
 - (e) -11
 - (f) None of the above.
19. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2$, $x_2 = 0$
 - (b) $x_1 = 1$, $x_2 = 0$
 - (c) $x_1 = 0$, $x_2 = 1$
 - (d) $x_1 = 0$, $x_2 = 0$
 - (e) $x_1 = 1$, $x_2 = 1$
 - (f) None of the above.

20. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:

- (a) $x \geq 0$
- (b) $y \geq 0$
- (c) $x + 2y \leq 3000$
- (d) $x + y \leq 2000$
- (e) $60x + 90y \leq 150,000$
- (f) None of the above.

21. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

22. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.

- (a) 1
- (b) 10
- (c) 20
- (d) 30
- (e) 0
- (f) None of the above.

23. Find the $(1, 2)$ entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

24. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- (f) None of the above.

25. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
- (b) 1
- (c) 5
- (d) 1/2
- (e) -1
- (f) None of the above.

Answer Key for Exam A

Complete the following exam and record your answers on the bubble sheet provided.

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Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

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$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

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16. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

(a) -8

(b) 10

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 - (c) $2x + 5y$
 - (d) $2x + 25y$
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18. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.
- (a) 0
 - (b) -17
 - (c) -5
 - (d) -1
 - (e) -11
 - (f) None of the above.
19. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2$, $x_2 = 0$
 - (b) $x_1 = 1$, $x_2 = 0$
 - (c) $x_1 = 0$, $x_2 = 1$
 - (d) $x_1 = 0$, $x_2 = 0$
 - (e) $x_1 = 1$, $x_2 = 1$
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20. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:

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- (e) $60x + 90y \leq 150,000$
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21. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

22. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.

- (a) 1
- (b) 10
- (c) 20
- (d) 30
- (e) 0
- (f) None of the above.

23. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

24. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
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- (f) None of the above.

25. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the (1,2) entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

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- (b) 1
- (c) 5
- (d) 1/2
- (e) -1
- (f) None of the above.

Complete the following exam and record your answers on the bubble sheet provided.

- Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
 - 0
 - 3
 - 6
 - 9
 - There is no solution to this problem.
 - None of the above.
- Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.
 - 6600
 - 2400
 - 5400
 - 900
 - 3300
 - None of the above.
- If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the (1, 2) entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- 2
- 1
- 5
- 1/2
- 1
- None of the above.

4. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.

- (a) $30x + 25y$
- (b) $x + 0.5y$
- (c) $2x + 5y$
- (d) $2x + 25y$
- (e) $30x + 0.5y$
- (f) None of the above.

5. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
 - (b) -1
 - (c) 4
 - (d) 1
 - (e) 6
 - (f) None of the above.
6. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.
- (a) 1
 - (b) 10
 - (c) 20
 - (d) 30
 - (e) 0
 - (f) None of the above.

7. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.

- (a) 4
- (b) 1
- (c) 5
- (d) 3
- (e) 2
- (f) None of the above.

8. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

- (a) 1
- (b) -1
- (c) -2
- (d) -3
- (e) 3
- (f) None of the above.

9. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ \hline -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) $(2, 1)$
- (b) $(2, 3)$
- (c) $(3, 1)$
- (d) $(3, 2)$
- (e) Pivoting is not the correct action for this tableau.
- (f) None of the above.

10. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?

- (a) BC
- (b) $A + C$
- (c) CA
- (d) AB
- (e) BB
- (f) None of the above.

11. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
 - (b) 10
 - (c) 1
 - (d) -1
 - (e) 8
 - (f) None of the above.
12. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2$, $x_2 = 0$
 - (b) $x_1 = 1$, $x_2 = 0$
 - (c) $x_1 = 0$, $x_2 = 1$
 - (d) $x_1 = 0$, $x_2 = 0$
 - (e) $x_1 = 1$, $x_2 = 1$
 - (f) None of the above.
13. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
- (b) 76
- (c) 12
- (d) 6
- (e) The objective does not increase.
- (f) None of the above.

14. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) (1, 10)
 - (b) (0, 12)
 - (c) (0, 0)
 - (d) (0, 11)
 - (e) (6, 0)
 - (f) None of the above.
15. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.
- (a) 0
 - (b) -17
 - (c) -5
 - (d) -1
 - (e) -11
 - (f) None of the above.
16. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:
- (a) $x \geq 0$
 - (b) $y \geq 0$
 - (c) $x + 2y \leq 3000$
 - (d) $x + y \leq 2000$
 - (e) $60x + 90y \leq 150,000$
 - (f) None of the above.

17. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
 - (b) 150
 - (c) 200
 - (d) 250
 - (e) 300
 - (f) None of the above.
18. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.
19. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccc|cc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
- (b) $(0, 3, 2)$
- (c) $(12, 20, 60)$
- (d) $(10, 0, 0)$
- (e) This is not a valid tableau.
- (f) None of the above.

20. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.
21. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
- (a) 700 steel, 900 lumber
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - (f) None of the above.
22. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
- (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (f) None of the above.

23. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
 - (b) 4
 - (c) 5
 - (d) 6
 - (e) 7
 - (f) None of the above.
24. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.
- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
 - (b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
 - (c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
 - (d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
 - (e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
 - (f) None of the above.
25. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

Answer Key for Exam B

Complete the following exam and record your answers on the bubble sheet provided.

1. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
 - (a) 0
 - (b) 3
 - (c) -6
 - (d) -9
 - (e) There is no solution to this problem.
 - (f) None of the above.

2. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.
 - (a) 6600
 - (b) 2400
 - (c) 5400
 - (d) 900
 - (e) 3300
 - (f) None of the above.

3. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
- (b) 1
- (c) 5
- (d) 1/2
- (e) -1
- (f) None of the above.

4. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.

- (a) $30x + 25y$
- (b) $x + 0.5y$
- (c) $2x + 5y$
- (d) $2x + 25y$
- (e) $30x + 0.5y$
- (f) None of the above.

5. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
 - (b) -1
 - (c) 4
 - (d) 1
 - (e) 6
 - (f) None of the above.
6. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.
- (a) 1
 - (b) 10
 - (c) 20
 - (d) 30
 - (e) 0
 - (f) None of the above.

7. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.

- (a) 4
- (b) 1
- (c) 5
- (d) 3
- (e) 2
- (f) None of the above.

8. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

- (a) 1
- (b) -1
- (c) -2
- (d) -3
- (e) 3
- (f) None of the above.

9. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) $(2, 1)$
- (b) $(2, 3)$
- (c) $(3, 1)$
- (d) $(3, 2)$
- (e) Pivoting is not the correct action for this tableau.
- (f) None of the above.

10. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?

- (a) BC
- (b) $A + C$
- (c) CA
- (d) AB
- (e) BB
- (f) None of the above.

11. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
(b) 10
(c) 1
(d) -1
(e) 8
(f) None of the above.
12. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2, x_2 = 0$
(b) $x_1 = 1, x_2 = 0$
(c) $x_1 = 0, x_2 = 1$
(d) $x_1 = 0, x_2 = 0$
(e) $x_1 = 1, x_2 = 1$
(f) None of the above.
13. After a certain number of steps using the simplex method for maximization, the tableau takes the form:
- $$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$
- After the next pivot, how much does the objective function increase?
- (a) 64
(b) 76
(c) 12
(d) 6
(e) The objective does not increase.
(f) None of the above.
14. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) $(1, 10)$
(b) $(0, 12)$
(c) $(0, 0)$
(d) $(0, 11)$
(e) $(6, 0)$
(f) None of the above.

15. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.

- (a) 0
- (b) -17
- (c) -5
- (d) -1
- (e) -11
- (f) None of the above.

16. Consider the problem: "A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?" Which of the following is not a constraint for this problem:

- (a) $x \geq 0$
- (b) $y \geq 0$
- (c) $x + 2y \leq 3000$
- (d) $x + y \leq 2000$
- (e) $60x + 90y \leq 150,000$
- (f) None of the above.

17. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- (f) None of the above.

18. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?

- (a) 7
- (b) 3
- (c) 2
- (d) 8
- (e) 3.5
- (f) None of the above.

19. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ \hline -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
 - (b) $(0, 3, 2)$
 - (c) $(12, 20, 60)$
 - (d) $(10, 0, 0)$
 - (e) This is not a valid tableau.
 - (f) None of the above.
20. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.
21. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
- (a) 700 steel, 900 lumber
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - (f) None of the above.

22. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.

- (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
- (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
- (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
- (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
- (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
- (f) None of the above.

23. Find the $(1, 2)$ entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

24. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.

- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
- (b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
- (c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
- (d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
- (e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
- (f) None of the above.

25. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

Complete the following exam and record your answers on the bubble sheet provided.

1. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
 - (b) -1
 - (c) 4
 - (d) 1
 - (e) 6
 - (f) None of the above.
2. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
 - (a) 700 steel, 900 lumber
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - (f) None of the above.
 3. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
 - (a) BC
 - (b) $A + C$
 - (c) CA
 - (d) AB
 - (e) BB
 - (f) None of the above.

4. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ \hline -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) (2, 1)
 - (b) (2, 3)
 - (c) (3, 1)
 - (d) (3, 2)
 - (e) Pivoting is not the correct action for this tableau.
 - (f) None of the above.
5. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
- (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (f) None of the above.

6. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
- (b) 10
- (c) 1
- (d) -1
- (e) 8
- (f) None of the above.

7. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
 - (b) 150
 - (c) 200
 - (d) 250
 - (e) 300
 - (f) None of the above.
8. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ \hline 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
 - (b) 76
 - (c) 12
 - (d) 6
 - (e) The objective does not increase.
 - (f) None of the above.
9. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1, x - y \geq 0, 3x + y - 4 \leq 0$.
- (a) 0
 - (b) -17
 - (c) -5
 - (d) -1
 - (e) -11
 - (f) None of the above.

10. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
- (a) $30x + 25y$
 - (b) $x + 0.5y$
 - (c) $2x + 5y$
 - (d) $2x + 25y$
 - (e) $30x + 0.5y$
 - (f) None of the above.
11. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.
- (a) 1
 - (b) 10
 - (c) 20
 - (d) 30
 - (e) 0
 - (f) None of the above.
12. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
- (a) 0
 - (b) 3
 - (c) -6
 - (d) -9
 - (e) There is no solution to this problem.
 - (f) None of the above.

13. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the (1, 2) entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
 - (b) 1
 - (c) 5
 - (d) $1/2$
 - (e) -1
 - (f) None of the above.
14. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.
15. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.
- (a) 4
 - (b) 1
 - (c) 5
 - (d) 3
 - (e) 2
 - (f) None of the above.

16. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.

- (a) 6600
- (b) 2400
- (c) 5400
- (d) 900
- (e) 3300
- (f) None of the above.

17. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

18. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

19. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:
- (a) $x \geq 0$
 - (b) $y \geq 0$
 - (c) $x + 2y \leq 3000$
 - (d) $x + y \leq 2000$
 - (e) $60x + 90y \leq 150,000$
 - (f) None of the above.
20. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.
- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
 - (b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
 - (c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
 - (d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
 - (e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
 - (f) None of the above.
21. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2, x_2 = 0$
 - (b) $x_1 = 1, x_2 = 0$
 - (c) $x_1 = 0, x_2 = 1$
 - (d) $x_1 = 0, x_2 = 0$
 - (e) $x_1 = 1, x_2 = 1$
 - (f) None of the above.

22. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ \hline -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
 - (b) $(0, 3, 2)$
 - (c) $(12, 20, 60)$
 - (d) $(10, 0, 0)$
 - (e) This is not a valid tableau.
 - (f) None of the above.
23. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.
24. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.
- (a) 1
 - (b) -1
 - (c) -2
 - (d) -3
 - (e) 3
 - (f) None of the above.
25. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) $(1, 10)$
 - (b) $(0, 12)$
 - (c) $(0, 0)$
 - (d) $(0, 11)$
 - (e) $(6, 0)$
 - (f) None of the above.

Answer Key for Exam C

Complete the following exam and record your answers on the bubble sheet provided.

1. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
 - (b) -1
 - (c) 4
 - (d) 1
 - (e) 6
 - (f) None of the above.
2. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
 - (a) 700 steel, 900 lumber
 - (b) 650 steel, 1000 lumber
 - (c) 140 steel, 180 lumber
 - (d) 130 steel, 200 lumber
 - (e) 800 steel, 700 lumber
 - (f) None of the above.
 3. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
 - (a) BC
 - (b) $A + C$
 - (c) CA
 - (d) AB
 - (e) BB
 - (f) None of the above.

4. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) (2, 1)
 - (b) (2, 3)
 - (c) (3, 1)
 - (d) (3, 2)
 - (e) Pivoting is not the correct action for this tableau.
 - (f) None of the above.
5. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
- (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (f) None of the above.
6. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
- (b) 10
- (c) 1
- (d) -1
- (e) 8
- (f) None of the above.

7. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- (f) None of the above.

8. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
- (b) 76
- (c) 12
- (d) 6
- (e) The objective does not increase.
- (f) None of the above.

9. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.

- (a) 0
- (b) -17
- (c) -5
- (d) -1
- (e) -11
- (f) None of the above.

10. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
- (a) $30x + 25y$
 - (b) $x + 0.5y$
 - (c) $2x + 5y$
 - (d) $2x + 25y$
 - (e) $30x + 0.5y$
 - (f) None of the above.
11. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.
- (a) 1
 - (b) 10
 - (c) 20
 - (d) 30
 - (e) 0
 - (f) None of the above.
12. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
- (a) 0
 - (b) 3
 - (c) -6
 - (d) -9
 - (e) There is no solution to this problem.
 - (f) None of the above.

13. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
 - (b) 1
 - (c) 5
 - (d) $1/2$
 - (e) -1
 - (f) None of the above.
14. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.
15. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.
- (a) 4
 - (b) 1
 - (c) 5
 - (d) 3
 - (e) 2
 - (f) None of the above.

16. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.

- (a) 6600
- (b) 2400
- (c) 5400
- (d) 900
- (e) 3300
- (f) None of the above.

17. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

18. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

19. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:

- (a) $x \geq 0$
- (b) $y \geq 0$
- (c) $x + 2y \leq 3000$
- (d) $x + y \leq 2000$
- (e) $60x + 90y \leq 150,000$
- (f) None of the above.

20. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.

- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
- (b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
- (c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
- (d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
- (e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
- (f) None of the above.

21. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .

- (a) $x_1 = 2, x_2 = 0$
- (b) $x_1 = 1, x_2 = 0$
- (c) $x_1 = 0, x_2 = 1$
- (d) $x_1 = 0, x_2 = 0$
- (e) $x_1 = 1, x_2 = 1$
- (f) None of the above.

22. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ \hline -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
 - (b) $(0, 3, 2)$
 - (c) $(12, 20, 60)$
 - (d) $(10, 0, 0)$
 - (e) This is not a valid tableau.
 - (f) None of the above.
23. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.
24. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.
- (a) 1
 - (b) -1
 - (c) -2
 - (d) -3
 - (e) 3
 - (f) None of the above.
25. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) $(1, 10)$
 - (b) $(0, 12)$
 - (c) $(0, 0)$
 - (d) $(0, 11)$
 - (e) $(6, 0)$
 - (f) None of the above.

Complete the following exam and record your answers on the bubble sheet provided.

1. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.

- (a) 1
- (b) 10
- (c) 20
- (d) 30
- (e) 0
- (f) None of the above.

2. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
 - (b) -4
 - (c) 6
 - (d) 4
 - (e) -5
 - (f) None of the above.
3. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
- (b) $(0, 3, 2)$
- (c) $(12, 20, 60)$
- (d) $(10, 0, 0)$
- (e) This is not a valid tableau.
- (f) None of the above.

4. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
(b) -1
(c) 4
(d) 1
(e) 6
(f) None of the above.
5. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.
- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
(b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
(c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
(d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
(e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
(f) None of the above.
6. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
- (a) 700 steel, 900 lumber
(b) 650 steel, 1000 lumber
(c) 140 steel, 180 lumber
(d) 130 steel, 200 lumber
(e) 800 steel, 700 lumber
(f) None of the above.

7. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
 - (b) 1
 - (c) 5
 - (d) $1/2$
 - (e) -1
 - (f) None of the above.
8. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.
9. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2, x_2 = 0$
 - (b) $x_1 = 1, x_2 = 0$
 - (c) $x_1 = 0, x_2 = 1$
 - (d) $x_1 = 0, x_2 = 0$
 - (e) $x_1 = 1, x_2 = 1$
 - (f) None of the above.

10. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
- (b) 10
- (c) 1
- (d) -1
- (e) 8
- (f) None of the above.

11. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- (f) None of the above.

12. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.

- (a) 0
- (b) -17
- (c) -5
- (d) -1
- (e) -11
- (f) None of the above.

13. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
(b) 4
(c) 5
(d) 6
(e) 7
(f) None of the above.
14. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
(a) BC
(b) $A + C$
(c) CA
(d) AB
(e) BB
(f) None of the above.
15. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
(a) 0
(b) 3
(c) -6
(d) -9
(e) There is no solution to this problem.
(f) None of the above.
16. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.
(a) 6600
(b) 2400
(c) 5400
(d) 900
(e) 3300
(f) None of the above.

17. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
- Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - None of the above.
18. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:
- $x \geq 0$
 - $y \geq 0$
 - $x + 2y \leq 3000$
 - $x + y \leq 2000$
 - $60x + 90y \leq 150,000$
 - None of the above.
19. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
- $30x + 25y$
 - $x + 0.5y$
 - $2x + 5y$
 - $2x + 25y$
 - $30x + 0.5y$
 - None of the above.

20. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.

21. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ \hline 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
 - (b) 76
 - (c) 12
 - (d) 6
 - (e) The objective does not increase.
 - (f) None of the above.
22. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.
- (a) 4
 - (b) 1
 - (c) 5
 - (d) 3
 - (e) 2
 - (f) None of the above.

23. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) (2, 1)
 - (b) (2, 3)
 - (c) (3, 1)
 - (d) (3, 2)
 - (e) Pivoting is not the correct action for this tableau.
 - (f) None of the above.
24. What is the (2, 1) entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.
- (a) 1
 - (b) -1
 - (c) -2
 - (d) -3
 - (e) 3
 - (f) None of the above.
25. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?
- (a) (1, 10)
 - (b) (0, 12)
 - (c) (0, 0)
 - (d) (0, 11)
 - (e) (6, 0)
 - (f) None of the above.

Answer Key for Exam D

Complete the following exam and record your answers on the bubble sheet provided.

1. Find the maximum value of $2x + y$ on the feasible set given by the constraints $x + 10y \leq 50$, $x + y \leq 5$, $10x + y \leq 50$, $x \geq 0$, $y \geq 0$.

- (a) 1
- (b) 10
- (c) 20
- (d) 30
- (e) 0
- (f) None of the above.

2. Suppose that x and y are numbers such that

$$\begin{pmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{pmatrix} = \begin{pmatrix} 9 & 9 \\ 18 & 2 \end{pmatrix}$$

Find x :

- (a) 2
- (b) -4
- (c) 6
- (d) 4
- (e) -5
- (f) None of the above.

3. A linear programming problem in standard maximum form has the initial tableau

$$\left[\begin{array}{cccccc|c} 1 & 4 & 4 & 1 & 0 & 0 & 16 \\ 2 & 1 & 5 & 0 & 1 & 0 & 20 \\ -3 & -1 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

What is the solution, written in terms of the variables (x_1, x_2, x_3) ?

- (a) $(0, 1, 2)$
- (b) $(0, 3, 2)$
- (c) $(12, 20, 60)$
- (d) $(10, 0, 0)$
- (e) This is not a valid tableau.
- (f) None of the above.

4. Kathy needed to find the inverse to a 3 by 3 matrix A , so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane for Thanksgiving break. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right]$$

Assuming Kathy was correct to this point, find the (1,2)-entry of A^{-1} .

- (a) -2
(b) -1
 (c) 4
(d) 1
(e) 6
(f) None of the above.
5. An economy has two goods: steel and grain. One needs 0.28 units of steel and 0.3 units of grain to produce 1 unit of steel, where as one needs 0.4 units of steel and 0.29 units of grain to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.
- (a) $\begin{pmatrix} 0.28 & 0.3 \\ 0.4 & 0.29 \end{pmatrix}$
(b) $\begin{pmatrix} 0.72 & -0.3 \\ -0.4 & 0.71 \end{pmatrix}$
(c) $\begin{pmatrix} 0.29 & 0.4 \\ 0.3 & 0.28 \end{pmatrix}$
 (d) $\begin{pmatrix} 0.28 & 0.4 \\ 0.3 & 0.29 \end{pmatrix}$
(e) $\begin{pmatrix} 0.72 & -0.4 \\ -0.3 & 0.71 \end{pmatrix}$
(f) None of the above.
6. In a steel-lumber economy it takes 0.2 units of steel and 0.5 units of lumber to produce one unit of steel, whereas it requires 0.6 units of steel and 0.6 units of coal to produce one unit of lumber. How many units of steel and coal must be produced to satisfy the external demand for 20 units of steel and 10 units of lumber?
- (a) 700 steel, 900 lumber
(b) 650 steel, 1000 lumber
(c) 140 steel, 180 lumber
(d) 130 steel, 200 lumber
(e) 800 steel, 700 lumber
(f) None of the above.

7. If A is a matrix with inverse

$$A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 5 & -4 \\ 1 & 2 & -2 \end{pmatrix}$$

find the $(1, 2)$ entry of the matrix X satisfying equation

$$XA = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

- (a) -2
 - (b) 1
 - (c) 5
 - (d) $1/2$
 - (e) -1
 - (f) None of the above.
8. Find the values y_1 and y_2 that minimize the objective $5y_1 + 2y_2$ subject to the constraints $y_1 \geq 0$, $y_2 \geq 0$, $2y_1 + 3y_2 \geq 6$, and $2y_1 + y_2 \geq 7$. What is the sum $y_1 + y_2$?
- (a) 7
 - (b) 3
 - (c) 2
 - (d) 8
 - (e) 3.5
 - (f) None of the above.
9. Consider the linear program: minimize $2y_1 + y_2 + 3y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 \geq 100$, and $2y_1 + y_2 \geq 50$. Solve the dual problem and report its solution variables x_1 and x_2 .
- (a) $x_1 = 2, x_2 = 0$
 - (b) $x_1 = 1, x_2 = 0$
 - (c) $x_1 = 0, x_2 = 1$
 - (d) $x_1 = 0, x_2 = 0$
 - (e) $x_1 = 1, x_2 = 1$
 - (f) None of the above.

10. Given

$$6 \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & x \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 22 & -8 \end{pmatrix}$$

Find x .

- (a) -8
- (b) 10
- (c) 1
- (d) -1
- (e) 8
- (f) None of the above.

11. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find: x_1 . [Recall: $X = AX + D$]

- (a) 100
- (b) 150
- (c) 200
- (d) 250
- (e) 300
- (f) None of the above.

12. Find the minimum value of the function $3x - 2y$ on the set $x \geq -1$, $x - y \geq 0$, $3x + y - 4 \leq 0$.

- (a) 0
- (b) -17
- (c) -5
- (d) -1
- (e) -11
- (f) None of the above.

13. Find the (1,2) entry in the matrix $2A - B$ where

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) None of the above.

14. Let A be a 2×3 matrix, B be a 3×3 matrix, and C a 3×2 matrix. Which of the following is not defined?
- (a) BC
 - (b) $A + C$
 - (c) CA
 - (d) AB
 - (e) BB
 - (f) None of the above.
15. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.
- (a) 0
 - (b) 3
 - (c) -6
 - (d) -9
 - (e) There is no solution to this problem.
 - (f) None of the above.
16. Find the maximum of $10x_1 + 15x_2 + 10x_3 + 5x_4$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_1 + x_2 + x_3 + x_4 \leq 300$, and $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 360$.
- (a) 6600
 - (b) 2400
 - (c) 5400
 - (d) 900
 - (e) 3300
 - (f) None of the above.
17. A woodcarver makes toy boats to sell at the Provo Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor, whereas a fancier tugboat requires 2 linear feet of wood and 90 minutes of labor. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the fair. The profit on the sailboat is \$12/boat and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. Formulate a linear programming problem to maximize profit.
- (a) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
 - (b) Maximize $18x + 12y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (c) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (d) Maximize $12x + 18y$, subject to $2x + 2y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
 - (e) Maximize $12x + 18y$, subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
 - (f) None of the above.

18. Consider the problem: “A farmer owns 2000 acres of land and can plant any combination of two crops: wheat and barley. Wheat takes a person-day of labor and costs \$90 for each acre, while an acre of barley takes 2 person-days of labor and costs \$60. Wheat produces \$170 of revenue per acre and barley produces \$190 of revenue per acre. Assume that the farmer has \$150,000 to spend and 3000 person-days of labor available; how many acres of wheat and barley, denoted x and y , respectively, should be planted to maximize revenue?” Which of the following is not a constraint for this problem:
- (a) $x \geq 0$
 - (b) $y \geq 0$
 - (c) $x + 2y \leq 3000$
 - (d) $x + y \leq 2000$
 - (e) $60x + 90y \leq 150,000$
 - (f) None of the above.
19. Consider the problem: “A nutritionist at the Medical Center has been asked to prepare a special diet for certain patients. She has decided that the meals should contain at least 400 mg of calcium, 10 mg of iron, and 40 mg of vitamin C. She has further decided that the meals are to be prepared from foods A and B. Each ounce of food A contains 30 mg of calcium, 1 mg of iron, 2 mg of vitamin C, and 2 mg of cholesterol. Each ounce of food B contains 25 mg of calcium, 0.5 mg of iron, 5 mg of vitamin C, and 5 mg of cholesterol. Find how many ounces of each type of food should be used in a meal so that the cholesterol content is minimized and the minimum requirements of calcium, iron and vitamin C are met.” Which of the following is the objective function when this situation is formulated as a linear programming problem: Let x be the number of ounces of food A and y be the number of ounces of food B.
- (a) $30x + 25y$
 - (b) $x + 0.5y$
 - (c) $2x + 5y$
 - (d) $2x + 25y$
 - (e) $30x + 0.5y$
 - (f) None of the above.
20. Find the values x_1 and x_2 that maximize the objective $5x_1 + 2x_2$ such that $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 4x_2 \leq 15$, and $3x_1 + x_2 \leq 10$. What is $x_1 - x_2$?
- (a) 0
 - (b) $5/2$
 - (c) 5
 - (d) $35/2$
 - (e) 2
 - (f) None of the above.

21. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccccc|c} 1 & 4 & 0 & 1 & 3 & 0 & 6 \\ 0 & 3 & 1 & 3 & 5 & 0 & 15 \\ \hline 0 & 5 & 0 & 1 & -6 & 1 & 64 \end{array} \right]$$

After the next pivot, how much does the objective function increase?

- (a) 64
 - (b) 76
 - (c) 12
 - (d) 6
 - (e) The objective does not increase.
 - (f) None of the above.
22. Which of the following represents the number of corner points of the feasible set determined by the inequalities $x \geq -2$, $y \geq -3$, $x + 2y \leq 10$, $2x + y \geq 10$.
- (a) 4
 - (b) 1
 - (c) 5
 - (d) 3
 - (e) 2
 - (f) None of the above.
23. After a certain number of steps using the simplex method for maximization, the tableau takes the form:

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 5 & 3 & 5 & 0 & 15 \\ \hline -2 & 5 & -6 & 1 & 64 \end{array} \right]$$

Which element of the tableau should you pivot on next?

- (a) (2, 1)
- (b) (2, 3)
- (c) (3, 1)
- (d) (3, 2)
- (e) Pivoting is not the correct action for this tableau.
- (f) None of the above.

24. What is the $(2, 1)$ entry of the inverse of $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$.

- (a) 1
- (b) -1
- (c) -2
- (d) -3
- (e) 3
- (f) None of the above.

25. A feasible set for a linear programming problem is defined by $x \geq 0$, $y \geq 0$, $2x + y \leq 12$, $4x + 4y \leq 44$. Which of the following is *not* a corner point of the feasible set?

- (a) $(1, 10)$
- (b) $(0, 12)$
- (c) $(0, 0)$
- (d) $(0, 11)$
- (e) $(6, 0)$
- (f) None of the above.