1. Prove that a bounded monotone sequence \( \{x_n\} \) of real numbers converges.

2. Suppose that \( f \) is a uniformly continuous mapping of a metric space \( X \) into a metric space \( Y \). Prove that \( \{f(x_n)\} \) is a Cauchy sequence in \( Y \) for every Cauchy sequence \( \{x_n\} \) in \( X \).

3. Let \( I = [0, 1] \) be the closed unit interval. Suppose \( f \) is a continuous mapping of \( I \) into \( I \). Prove that \( f(x) = x \) for at least one \( x \in I \).

4. Prove or disprove: for \( A \subseteq \mathbb{R}^n \), if \( F : A \to \mathbb{R}^m \) is continuous and \( A \) is open, then \( F(A) \) is open.

5. Let \( f \) be a real-valued function defined on \([a, b]\). If \( f \) has a local maximum at a point \( c \in (a, b) \) and if \( f'(c) \) exists, prove that \( f'(c) = 0 \).

6. State the Mean Value Theorem for a continuously differentiable function \( f \) on \( \mathbb{R}^n \).

7. Let \( D \) be a (compact) Jordan domain in \( \mathbb{R}^k \). If a sequence of continuous functions \( \{f_n\} \) on \( D \) converges uniformly to \( f \), prove that (a) \( f \) is integrable on \( D \), and (b) that

\[
\lim_{n \to \infty} \int_D f_n = \int_D f.
\]

8. Let \( f \) be a continuously differentiable function on a simply connected open subset \( U \) of \( \mathbb{R}^n \). Prove for any rectifiable simple closed curve \( \gamma \) in \( U \) that

\[
\int_{\gamma} \langle \nabla f, d\gamma \rangle = 0,
\]

where \( \langle x, y \rangle \) is the standard inner product in \( \mathbb{R}^n \).

9. Find a linear transformation of \( \mathbb{C} \) which carries \( |z| = 1 \) and \( |z - 1/4| = 1/4 \) into concentric circles.

10. Find the value of

\[
\sum_{n=1}^{\infty} \frac{1}{n^4}.
\]