Name:__________________________________________

Student ID (see bubble sheet):____________________

Section:________________________________________

Instructor:_____________________________________  

Math 113 (Calculus II)  
Exam 2 Part B  
Mar. 5, 6. Mar. 7 Late Day  

TWO PART

Instructions:

• For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.

• Simplify your answers.

• Calculators are not allowed.

• Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.

• Please do not talk about the test with other students until after the last day to take the exam.

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<tr>
<th>#</th>
<th>Possible</th>
<th>Earned</th>
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<tbody>
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Part B: Short Answer. Evaluate or give the best response in the blank provided. Work will not be graded in this section. Only the answer will be graded. Questions are worth 2 points each.

10. (12 points)
   a) \[ \int \frac{1}{x \sqrt{\ln x}} \, dx = \]

   b) \[ \int_{0}^{2} \ln(x) \, dx = \]

   c) Set up the integral (but do not evaluate it) that gives the arc length of the curve \( xy = 2 \) from (1, 2) to (2, 1)

   d) Three functions \( f, g, \) and \( h \) are defined to have the value zero outside \([0, 1]\). Circle each function that gives a probability density function, and cross out each function that does not.

   \[
f(x) = x \quad \text{on} \quad [0, 1]; \quad g(x) = 2x \quad \text{on} \quad [0, 1]; \quad \text{and} \quad h(x) = 3x - 1 \quad \text{on} \quad [0, 1].
\]

   e) The function \( f(x) = xe^{-x} \), for \( x \geq 0 \), and \( f(x) = 0 \) for \( x < 0 \) is a probability density function. Find \( P(2 \leq X \leq 3) \).

   f) Set up the integral (but do not evaluate it) for the surface area of the surface obtained by rotating the curve \( y = 1 + 2x^2 \), \( 1 \leq x \leq 4 \) about the \( x \)-axis.
Part C: Show all work in the space provided.

11. (12 points)

a. (2 points) Let $C$ be the curve $y = 1/x$ for $1 \leq x \leq 5$. Set up an integral (but do not evaluate it) that expresses the arc length of $C$.

b. (3 points) Let $S$ be the surface obtained by rotating $C$ about the $x$-axis. Set up an integral that expresses the surface area of $S$.

c. (2 points) A surface $G$ (called Gabriel’s horn) is obtained by rotating the curve $y = 1/x$ for $1 \leq x \leq \infty$ about the $x$-axis. Write an improper integral that expresses the surface area of $G$.

d. (3 points) Use the Comparison Theorem to show that your answer to part c is divergent.

e. (2 points) An improper integral is defined as a limit (in this case, a limit that diverges). Write the answer to part c as a limit.
12. (16 points)
A car applies its the brakes at time \( t = 0 \) and at half-second intervals its velocity \( v(t) \) is measured in meters per second. The results are given in the table below.

a. (2 points) Use the trapezoid rule with \( n = 4 \) to estimate the total braking distance.

<table>
<thead>
<tr>
<th>time (s)</th>
<th>speed (m/s)</th>
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</thead>
<tbody>
<tr>
<td>0.0</td>
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<tr>
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<td>9</td>
</tr>
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<tr>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
</tr>
</tbody>
</table>

b. (4 points) The derivative of acceleration \( v''(t) \) is sometimes called the \textit{jerk}. Assuming that \( |v''(t)| \) during braking never exceeded 20 m/s\(^3\), and using the trapezoid rule error bound

\[
|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{for} \quad |v''(x)| \leq K,
\]

give a maximum possible error for your estimate in part a.

c. (2 points) Use the midpoint rule with \( n = 2 \) to estimate the total braking distance.

d. (4 points) Under the same assumptions as part b and using the midpoint rule error bound

\[
|E_M| \leq \frac{K(b-a)^3}{24n^2} \quad \text{for} \quad |v''(x)| \leq K,
\]

give a maximum possible error for your estimate in part c.
e. (4 points) Use Simpson’s Rule with $n = 4$ to estimate the total braking distance.

13. (6 points) The curve $y = \sqrt{9 - x^2}, -2 \leq x \leq 2$, is an arc of the circle $x^2 + y^2 = 9$. Find the area of the surface obtained by rotating this arc about the $x$-axis.
14. (6 points) A cubical tank of height 5 meters is filled with a fluid of density 2000 kg/m³. Calculate the hydrostatic force on one vertical square face of the tank. (Acceleration due to gravity is 9.8 m/s².)

15. (6 points) (a) Find the centroid of a system with three particles: a particle of mass 1 at point (0, 0), a particle of mass 2 at point (2, 0), and a particle of mass 6 at point (0, 3).

(b) Find the centroid of the region bounded by the curves $y = x^2$ and $y = 4x$. 
16. (6 points) Find \[ \int_{0}^{\infty} \frac{x^2}{9 + x^6} \, dx. \] (You may want to use the substitution \( u = x^3 \) and write the improper integral as a limit.)