Math 118

DEPARTMENTAL EXAM 1
FORM A

This exam is composed of 25 multiple choice questions each worth 4 points.

There is no time limit.

No notes are allowed.

Please do not write on the exam.

Samuel Sherman
Math 118 Exam 1

1. In a certain community, 32 percent of families own a Toyota, and 20 percent of families that own a Ford also own a Toyota. In addition, 35 percent of families own a Ford. What is the probability that a randomly selected family owns both a Toyota and a Ford?

(a) 0.064
(b) 0.070
(c) 0.112
(d) 0.253
(e) 0.402
(f) None of the above.

2. How many 7-symbol passwords can be formed using the symbols \{1, 2, 3, a, b, c, d, e\}, if every password is required to have a number? (Note: Repeated symbols are allowed in passwords.)

(a) \(7 \cdot P(8, 5)\)
(b) \(\frac{8^7}{P(8, 5)}\)
(c) \(8^7 - 5^7\)
(d) \(5^5 + 5^4 + 5^3\)
(e) \(P(8, 7) - P(8, 5)\)
(f) None of the above.

3. If \(n(A) = 29\), \(n(B) = 35\), \(n(C) = 26\), \(n(A \cap B) = 15\), \(n(B \cap C) = 16\), \(n(A \cap C) = 12\), \(n(A \cap B \cap C) = 10\), find the number of elements in \((A \cap B)' \cap C\).

(a) 2
(b) 5
(c) 8
(d) 12
(e) 16
(f) None of the above.
4. Using the information of the table, find $P(E|F)$.

<table>
<thead>
<tr>
<th></th>
<th>English Major</th>
<th>Accounting Major</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(E)</td>
<td>(A)</td>
<td>(O)</td>
<td></td>
</tr>
<tr>
<td>Freshman (F)</td>
<td>20</td>
<td>15</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>Sophomore (S)</td>
<td>20</td>
<td>40</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>Junior (J)</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>Senior (R)</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>54</strong></td>
<td><strong>110</strong></td>
<td><strong>36</strong></td>
<td><strong>200</strong></td>
</tr>
</tbody>
</table>

(a) $\frac{1}{2}$  
(b) $\frac{1}{10}$  
(c) $\frac{1}{5}$  
(d) $\frac{1}{8}$  
(e) $\frac{2}{5}$  
(f) None of the above.

Find probability of an English major, given they are a freshman.

\[
\frac{20}{40} = \frac{1}{2}
\]

5. A health inspection is being conducted on three-fast food chains in Sunnydale, CA: Doublemeat Palace, Burger Queen, and Angel's Burger Emporium. The following table gives the number of each type of fast-food restaurant in Sunnydale, as well as the probability of passing the inspection. If a randomly chosen restaurant fails the inspection, find the probability that it is one of the Doublemeat Palace restaurants.

<table>
<thead>
<tr>
<th>Fast-Food Chain</th>
<th>Number of Restaurants</th>
<th>Probability of Passing the Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doublemeat Palace</td>
<td>5</td>
<td>0.40</td>
</tr>
<tr>
<td>Burger Queen</td>
<td>3</td>
<td>0.80</td>
</tr>
<tr>
<td>Angel's Burger Emporium</td>
<td>2</td>
<td>0.95</td>
</tr>
</tbody>
</table>

(a) $\frac{20}{63}$  
(b) $\frac{4}{29}$  
(c) $\frac{6}{37}$  
(d) $\frac{30}{37}$  
(e) $\frac{10}{31}$  
(f) None of the above.
6. At a certain high school, all students must take exactly one foreign language class. The foreign languages offered at that school are Spanish, French, and German. A total of 52 percent of students take Spanish, 35 percent of students take French, and 13 percent of students take German. The school took a survey and found that 40 percent of the students taking Spanish had visited a foreign country, 20 percent of the students taking French had visited a foreign country, and 30 percent of the students taking German had visited a foreign country. A student is chosen at random. Given that this person has visited a foreign country, what is the probability that he or she is taking German?

(a) 39/317
(b) 70/317
(c) 208/317
(d) 332/317
(e) 20/317
(f) None of the above.

\[ \frac{(0.13)(0.30)}{(0.52)(0.40) + (0.35)(0.20) + (0.13)(0.30)} = \frac{0.039}{\frac{311}{311}} = \frac{39}{311} \]

7. In a certain high school's math club, there are 10 freshmen, 6 sophomores, 4 juniors, and 3 seniors. Eight members of the club are to be sent to a math competition, with two club members chosen from each school year. How many different groups of students can be sent?

(a) \( \frac{C(23,8)}{C(10,2)C(6,2)C(4,2)C(3,2)} \)
(b) \( C(23,2)^4 \)
(c) \( C(10,2)C(6,2)C(4,2)C(3,2) \)
(d) \( \frac{C(23,8)}{10!6!4!3!} \)
(e) \( C(23,8) \)
(f) None of the above.

8. A quiz consists of 3 multiple choice questions with 4 possible answers each. If a student guesses the answer to each question, what is the probability that they get all the answers correct?

(a) \( \frac{1}{3} \)
(b) \( \frac{1}{4} \)
(c) \( \frac{1}{4^3} \)
(d) \( \frac{1}{C(4,3)} \)
(e) \( \frac{1}{4^3} \)
(f) None of the above.
9. How many ways can the letters in the word MISSISSIPPI be arranged if the first letter must be an I?

(a) \( \frac{11!}{1!3!4!2!} \)
(b) \( \frac{11!}{1!4!4!2!} \)
(c) \( C(11, 10) \)
(d) 10!
(e) \( \frac{10!}{1!4!3!2!} \)
(f) None of the above

10. Two fair, six sided dice are rolled. What is the probability that they show the same number?

(a) \( \frac{1}{6} \)
(b) \( \frac{5}{9} \)
(c) \( \frac{1}{3} \)
(d) \( \frac{10}{36} \)
(e) \( \frac{5}{6} \)
(f) None of the above

11. Let \( S \) be a probability space with \( A, B \subset S \). Suppose that \( P(A \cap B) = 0.2 \), \( P(A \cup B) = 0.7 \), and \( P(A) = 0.4 \). What is \( P(B) \)?

(a) 0.4
(b) 0.5
(c) 0.9
(d) 0.7
(e) 0.3
(f) None of the above

12. Let \( A \) and \( B \) be independent events. If \( P(A) = \frac{1}{2} \) and \( P(B) = \frac{1}{3} \), find \( P(A \cap B) \) and \( P(A \cup B) \).

(a) \( P(A \cap B) = \frac{2}{3}; P(A \cup B) = \frac{1}{6} \)
(b) \( P(A \cap B) = \frac{1}{6}; P(A \cup B) = \frac{1}{2} \)
(c) \( P(A \cap B) = \frac{1}{3}; P(A \cup B) = \frac{2}{3} \)
(d) \( P(A \cap B) = \frac{1}{6}; P(A \cup B) = 1 \)
(e) \( P(A \cap B) = \frac{1}{3}; P(A \cup B) = \frac{1}{2} \)
(f) None of the above
13. If you are dealt 4 cards from a standard deck, what is the probability that exactly 3 cards are of the same suit?

(a) \[4 \cdot C(13, 3) \cdot 3 \cdot C(13, 1)\]
(b) \[\frac{4! \cdot 4 \cdot 13 \cdot 12 \cdot 11 \cdot 39}{52 \cdot 51 \cdot 50 \cdot 49}\]
(c) \[\frac{C(13, 3) \cdot 3 \cdot C(13, 1)}{C(52, 4)}\]
(d) \[\frac{4 \cdot C(13, 3) \cdot C(13, 1)}{C(52, 4)}\]
(e) \[\frac{4 \cdot C(13, 3) \cdot 3 \cdot C(13, 1)}{C(52, 4)}\]
(f) None of the above.

14. Find \(C(7, 3)\)

(a) 35
(b) 720
(c) 840
(d) 1050
(e) 1260
(f) None of the above.

\[
\frac{7!}{3! \cdot 4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}, \quad \frac{210}{6} = 35
\]
15. Which of the sets below corresponds to the shaded area of the given Venn Diagram?

(a) \( A \cap B' \)
(b) \( (A \cup B') \cap C \)
(c) \( (A' \cap B') \cap C \)
(d) \( A \cap (B' \cap C) \)
(e) \( B' \cap C \)
(f) None of the above.

16. An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.

(ii) 70% of customers insure more than one car.

(iii) 20% of the customers insure a sports car.

(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected driver insures exactly one car and that car is not a sports car.

(a) 0.13
(b) 0.205
(c) 0.24
(d) 0.25
(e) 0.30
(f) None of the above.

15. Which of the sets below corresponds to the shaded area of the given Venn Diagram?

(a) \( A \cap B' \)
(b) \( (A \cup B') \cap C \)
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(d) \( A \cap (B' \cap C) \)
(e) \( B' \cap C \)
(f) None of the above.

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(a) 0.13
(b) 0.205
(c) 0.24
(d) 0.25
(e) 0.30
(f) None of the above.
17. Annie needs to staff the ring toss booth for a carnival. She has 10 employees who are willing to help at the booth. If she needs one employee to be at the booth for each of three time spots, in how many ways can she choose employees to staff the booth? (Assume an employee can only staff the booth for one time slot.)

\[ 10 \cdot 9 \cdot 8 = 720 \]

(a) 120
(b) 1000
(c) 5040
(d) 650
(e) 720
(f) None of the above.

18. In a high school of 800 students, 120 students are in choir, 75 students are in band, and 30 students are in both choir and band. How many students are in band or in choir, but not both?

\[ \frac{120}{30} \cdot \frac{75}{30} \]

\[ 90 + 45 = 135 \]

(a) 135
(b) 90
(c) 125
(d) 45
(e) 165
(f) None of the above.

19. Rachel visits her hair stylist to get her hair cut and colored. She is trying to decide between 4 shades of red for the color and 3 styles of short hair cuts. How many different hairstyle options does Rachel have?

(a) 12
(b) 4
(c) 3
(d) \(4^3\)
(e) \(3^4\)
(f) None of the above.
20. A salesman has six prospects, including a person in Provo. If he randomly arranges his schedule to visit four of the six prospects, find the probability that the customer from Provo is NOT visited.

(a) $\frac{1}{6}
(b) \frac{1}{9}
(c) \frac{1}{5}
(d) \frac{1}{2}
(e) \frac{1}{3}
(f) None of the above.

21. Ginny has 5 pairs of shoes (10 shoes total) in her closet. If she reaches in and grabs two shoes at random, what is the probability that she gets a matching pair?

(a) $\frac{1}{10}$
(b) $\frac{1}{9}$
(c) $\frac{1}{5}$
(d) $\frac{1}{2}$
(e) $\frac{1}{3}$
(f) None of the above.

22. An experiment has possible sample space $S = \{s_1, s_2, s_3, s_4\}$. If $P(s_1) = \frac{1}{2}$, $P(s_2) = \frac{1}{6}$, and $P(s_3) = \frac{1}{3}$, what can be said about $P(s_4)$?

(a) $P(s_4) = \frac{1}{5}$
(b) $P(s_4) = 0$
(c) $P(s_4) = -\frac{1}{3}$
(d) There is not enough information to determine $P(s_4)$.
(e) This is not a valid assignment of probabilities.
(f) None of the above.

23. Matthew and Caroline are at the grocery store buying toppings for an ice cream sundae. They already know that they want three different toppings on their vanilla ice cream. If there are 8 toppings available for purchase, how many different sundaes can they make?

(a) 210
(b) 24
(c) 336
(d) 56
(e) 512
(f) None of the above.
24. A dentist has 8 patients in the waiting room, 2 men and 6 women. The probability that a male patient has a cavity is 40%, whereas the probability that a female patient has a cavity is 20%. Given that a patient has a cavity, what is the probability that the patient is a male?

(a) 2/5
(b) 1/15
(c) 1/16
(d) 3/16
(e) 1/10
(f) None of the above.

\[
\frac{\left(\frac{2}{8}\right)\left(\frac{1}{2}\right)}{\left(\frac{2}{8}\right)\left(\frac{1}{2}\right) + \left(\frac{6}{8}\right)\left(\frac{1}{5}\right)} = \frac{1}{2.5} = 0.4
\]

25. When rolling 3 fair 6-sided dice, what is the probability of rolling at least 2 different numbers?

(a) \(\frac{35}{36}\)
(b) \(\frac{5}{6}\)
(c) \(\frac{1}{6}\)
(d) \(\frac{1}{36}\)
(e) \(\frac{5}{36}\)
(f) None of the above.

1 - probability of rolling the same number

1 - \(\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\)

1 - \(\frac{1}{36}\)

\(\frac{35}{36}\)