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Exam 3 Review

Math 118 Sections 1 and 2

This exam will cover sections 5.3-5.6, 6.1-6.3 and 7.1-7.3 of the textbook. No books, notes, calculators or other aids are allowed on this exam. There is no time limit. It will consist of 25 multiple choice questions. All exam questions will have the option 'None of the above', but when writing the exam, we intend to always include the correct answer among the other options. This means 'None of the above' will be a correct answer only if there is a typo or similar type mistake.

It is important that you know the terms, notations and formulas in the book. There are a disproportionate number of basic questions on this review, because almost all problems build on the knowledge of basic information. Some of the exam problems will mimic homework problems, but roughly 20% of exam questions will be what I call extension questions. See the last page section for a description and examples of extension questions.

The review questions included are not all possible types of questions. They are just representative of the types of questions that could be asked to test the given concept(s).

The following are the concepts which will be tested on this exam.

A.1 Know how to determine the size of a matrix, and be familiar with the terminology associated with matrices: row, column, square matrix, row matrix, column matrix, row vector, column vector, identity matrix (p.255), invertible matrix, inverse matrix, zero matrix (p.268). Know the definition of matrix equality (p.239). Know when two matrices can be added and subtracted and how to add and subtract matrices. (5.3)

A.2 Be able to multiply a scalar and a matrix. Know when two matrices can be multiplied and how to multiply matrices.(5.4) $(r \times \underbrace{c)(r} \times c)$

A.3 Know the characteristics of an identity matrix. Know the definition and characteristics of an inverse matrix. Be able to calculate the inverse of a 2×2 or 3×3 matrix. (5.5)

A.4 NOTE: You need to be comfortable with row operations and the Gauss Jordan method for this exam.

A.5 NOTE: The $(2,3)$ entry of a matrix is the entry in the second row and third column.

Question 1-5 refer to the matrices $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{bmatrix}$

$D = \begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix}$.

1. Find $A^T + B$, if possible. ✓

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 & 4 \\ 2 & 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 \\ 0 & 4 \\ 4 & 5 \end{bmatrix}$

(e) They cannot be added because they are the wrong sizes.

(f) None of the Above.

2. Find $BA - 3D$, if possible.

(a) $\begin{bmatrix} 6 & 7 \\ -2 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 1 \\ -12 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 19 \\ 18 & -1 \end{bmatrix}$

(d) The matrix BA cannot be calculated.

(e) BA and $3D$ cannot be subtracted.

(f) None of the Above.

3. Find the $(3,1)$ entry of CB, if possible.

(a) 39

(b) 5

(c) 4

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 0 & 4 \\ 39 & 52 \end{bmatrix}$$

3×2 $\quad$ 3×2

(d) CB has no $(3,1)$ entry.

(e) C and B cannot be multiplied.

(f) None of the Above.

4. Find A^{-1}, if possible.

(a) $\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

(d) $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

(e) A has no inverse.

(f) None of the Above.

5. Find the $(2,2)$ entry of D^{-1}, if possible.

(a) -2

(b) $2/13$

(c) $1/13$

$$\left[\begin{array}{cc|cc} -2 & 3 & 1 & 0 \\ 5 & -1 & 0 & 1 \end{array}\right] 5R_1 + 2R_2$$

$$\left[\begin{array}{cc|cc} -2 & 3 & 1 & 0 \\ 0 & 13 & 5 & 2 \end{array}\right] 3R_2 - 13R_1$$

(d) $-1/13$

(e) D has no inverse.

(f) None of the Above.

$$\left[\begin{array}{cc|cc} 26 & 0 & 2 & 6 \\ 0 & 13 & 5 & 2 \end{array}\right]$$

$$\begin{bmatrix} 1/13 & 3/13 \\ 5/13 & 2/13 \end{bmatrix}$$

6. If A is a 2×3 matrix and C is a 2×2 matrix, and $AB = C$, what size is B?

(a) 2×2

(b) 2×3

(c) 3×2

(d) 3×3

$$\begin{bmatrix} X & X & X \\ X & X & X \end{bmatrix}\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} X & X \\ X & X \end{bmatrix}$$

$3 \times$

$R \times C$

(e) There is no matrix B for which this can happen.

(f) None of the Above.

7. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{bmatrix}$. Which of the following is true about A^{-1}?

(a) 3 occurs as the $(1,2)$ entry of A^{-1}.

(b) 3 occurs as the $(2,3)$ entry of A^{-1}.

(c) 3 occurs as the $(3,1)$ entry of A^{-1}.

(d) 3 occurs as the $(3,2)$ entry of A^{-1}.

(e) 3 does not occur as an entry of A^{-1}.

(f) None of the Above.

$$A^{-1} = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{bmatrix}$$

A.6 Know how to write a linear system as a matrix equation, and know how to use an inverse matrix to solve a matrix equation (p.259). Be comfortable solving a system with the Gauss Jordan method or using the inverse matrix, as appropriate. (5.5)

A.7 Know the terminology associated with Leontief input-output models: input-output matrix, production matrix, demand matrix, open model, closed model. In particular, know that the input-output matrix of a Leontief input-output model is sometimes called the *technology matrix*. Be able to answer questions related to Leontief input-output model. In particular, be able solve an open model or closed model in two or three variables for the production matrix. (5.6)

8. If the linear system $\left\{ \begin{array}{rcl} 3x & - & 6y = 7 \\ -2x & + & 5y = 12 \end{array} \right\}$ is written as a matrix equation $AX = B$, find A.

(a) $A = \begin{bmatrix} 3 & -6 \\ -2 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$

(c) $A = \begin{bmatrix} -6 & 7 \\ 5 & 12 \end{bmatrix}$

(d) $A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(e) $A = \begin{bmatrix} \frac{50}{3} \\ \frac{107}{3} \end{bmatrix}$

(f) None of the Above.

$AB = C$
$B = A^{-1}C$
$B \neq CA^{-1}$

9. If $A = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$, and $AX = B$, find x. $A^{-1}B = X$

(a) $x = -2$

(b) $x = -1$

(c) $x = 1$

(d) $x = 2$

(e) $x = 3$

(f) None of the Above.

$\begin{bmatrix} 1/4 & 1/4 \\ -1/8 & 3/4 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

10. A simple economy depends on three commodities: agriculture, manufacturing, and households (i.e. the sector of the economy that produces labor). Production of one unit of agriculture requires 0.25 units of agriculture, 0.14 units of manufacturing, and 0.80 units of households. To produce 1 unit of manufacturing requires 0.40 units of agriculture, 0.12 units of manufacturing, and 3.60 units of households. To produce one unit of household requires 1.33 units of agriculture, 0.100 units of manufacturing, and 0.133 units of households. Find the technology matrix A.

↑↓

$(I - A)X = D$

(a) $A = \begin{bmatrix} 0.25 & 0.40 & 1.33 \\ 0.14 & 0.12 & 0.1 \\ 0.80 & 3.6 & 0.133 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0.25 & 0.14 & 0.80 \\ 0.40 & 0.12 & 3.6 \\ 1.33 & 0.1 & 0.133 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0.75 & -0.40 & -1.33 \\ -0.14 & 0.88 & -0.1 \\ -0.80 & -3.6 & 0.867 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0.75 & -0.14 & -0.80 \\ -0.40 & 0.88 & -3.6 \\ -1.33 & -0.1 & 0.867 \end{bmatrix}$

(e) There is not enough information to find the technology matrix.

(f) None of the Above.

Closed Economy

$D = 0$

Set z = Param

11. If a Leontief input-output model has technology matrix $A = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$, demand matrix $D = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$, and the production matrix is $X = \begin{bmatrix} x \\ y \end{bmatrix}$, find x. (The solution does not need to be in whole units.)

$X = AX + D$ $\qquad (I-A)^{-1}D = X$

$= \begin{bmatrix} 1.4583 & .20833 \\ .20833 & 1.4583 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 12.5 \\ 7.5 \end{bmatrix}$

(a) 2.5
(b) 5
(c) 7.5
(d) 10
(e) 12.5
(f) None of the Above.

12. A closed economy has technology matrix $A = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 1/4 & 1/4 \\ 1/3 & 1/4 & 3/4 \end{bmatrix}$. If the ratio of products is $x : y : z$, find $\frac{x}{z}$.

$\frac{x}{z}$ $\quad (I-A)X = D = 0$

$\begin{bmatrix} 2/3 & -1/2 & 0 \\ -1/3 & 3/4 & -1/4 \\ -1/3 & -1/4 & 1/4 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\frac{3}{8} : \frac{1}{2} : 1$

$\cancel{\frac{3/8}{1}} = \boxed{3/8}$

$y = \frac{1}{2}z$
$x = \frac{3}{8}z$
$z = z$

(a) $\frac{1}{8}$
(b) $\frac{3}{8}$
(c) $\frac{5}{8}$
(d) $\frac{7}{8}$
(e) $\frac{9}{8}$
(f) None of the Above.

13. If a Leontief input-output model has technology matrix $A = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$, and production is to be $X = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$, what demand can be met?

$\begin{bmatrix} .7 & -.1 \\ -.1 & .7 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$

$(I-A)^{-1}D = X$
$(I-A)X = D$

(a) $D = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$
(b) $D = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$
(c) $D = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$
(d) $D = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$
(e) $D = \begin{bmatrix} 12.5 \\ 12.5 \end{bmatrix}$
(f) None of the Above.

A.8 Know how to graph a linear inequality and systems of linear inequalities, and be familiar with the terminology of linear inequalities: half plane, boundary, solution, feasible region. (6.1)

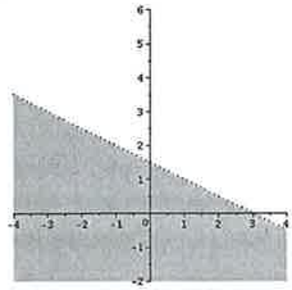
A.9 Be able to identify a linear programming problem and know the terminology associated with linear programming: objective function, constraints, corner point. Know the statement of the Corner Point Theorem, and how to apply it to solve a linear programming problem (p.290).

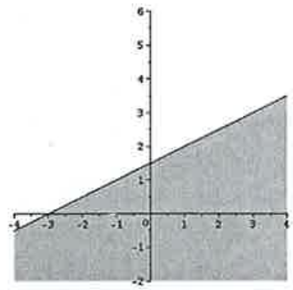
A.10 Be able to set up and solve a linear programming problem in two variables. (6.2, 6.3)

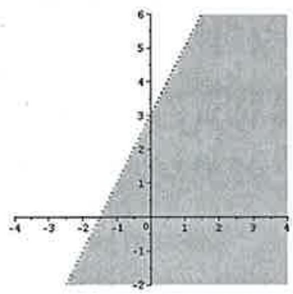
14. Which graph below is the graph of the inequality $3 < 2y - x$?

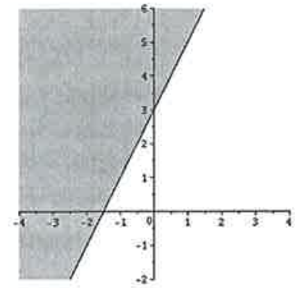
$2y > x + 3$

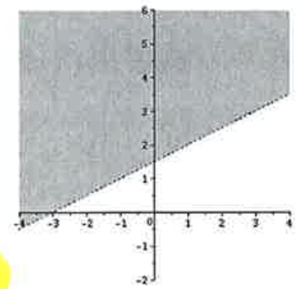
$y > \frac{1}{2}x + \frac{3}{2}$

(a) 

(b) 

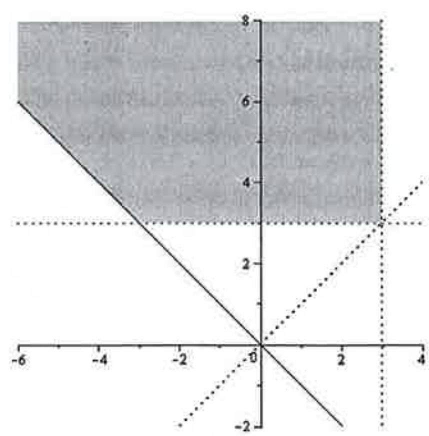
(c) 

(d) 

(e) 

(f) None of the Above.

15. Which set of constraints listed below corresponds to the given feasibility region?



$y > 3$

$y > x$

$x < 3$

$y \geq -x$

~~(a)~~ $0 < x - y, 0 > y + x, 3 > y, x < 3$

~~(b)~~ $0 \leq x - y, 0 \leq y + x, 3 > y, x < 3$

~~(c)~~ $0 \geq y - x, 0 < y + x, 3 \leq y, x \geq 3$

(d) $0 < y - x, 0 \leq y + x, 3 < y, x < 3$

~~(e)~~ $0 > x - y, 0 \leq y + x, 3 < y, x \geq 3$

(f) None of the Above.

16. Which statement below is most correct about the feasibility region corresponding to the constraints: $0 \leq x - y, 0 \leq y + x, 3 \geq y, x \leq 3$

$y \leq x$ $y \geq -x$ $y \leq 3$ $x \leq 3$

(a) This region is empty.

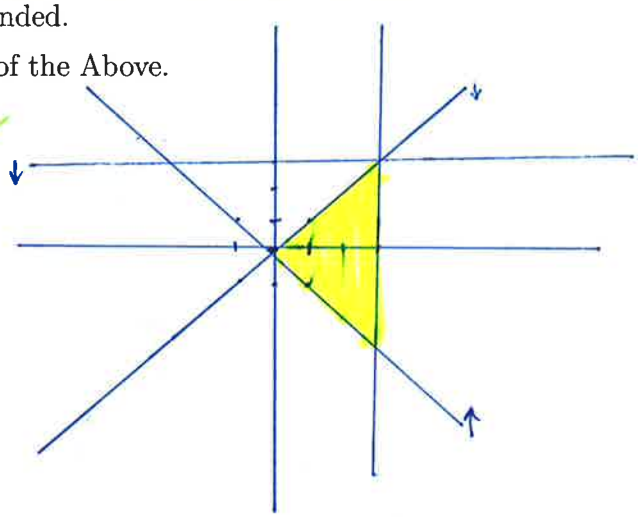
(b) This region has 2 corner points and is bounded.

(c) This region has 2 corner points and is unbounded.

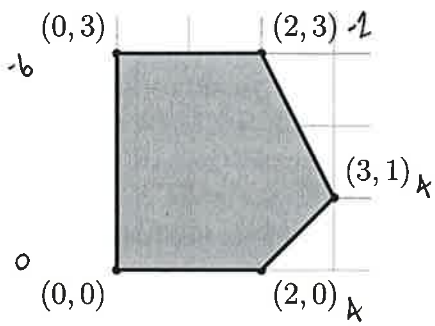
(d) This region has 3 corner points and is bounded.

(e) This region has 4 corner points and is unbounded.

(f) None of the Above.



17. Maximize $z = 2x - 2y$ on the given feasibility region.



(a) Maximum is 0 at $(0, 0)$

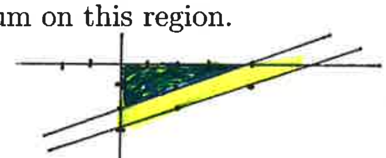
(b) Maximum is 4 at $(2, 0)$

(c) Maximum is 4 at $(2, 0)$ and $(3, 1)$ and all points in between.

(d) Maximum is 10 at $(2, 3)$

(e) There is no maximum on this region.

(f) None of the Above.



18. Minimize $z = 2x - 2y$ subject to $\left\{\begin{array}{rcrcr} x & - & 2y & \leq & 6 \\ x & - & 2y & \geq & 4 \\ & & x & \geq & 0 \\ & & y & \leq & 0 \end{array}\right\}$. $y \geq \frac{1}{2}x - 3$, $y \leq \frac{1}{2}x - 2$

(0,-2) = +4
(0,-3) = 6
(4, 0) = 8
(6, 0) = 12

(a) Minimum is 4.

(b) Minimum is 3.

(c) Minimum is 0.

(d) Minimum is −3.

(e) There is no minimum on this region.

(f) None of the Above.

19. Consider the problem: "A candy company has 150 kg of chocolate-covered nuts and 90 kg of chocolate-covered raisins to be sold as two different mixes. The first mix will contain half nuts and half raisins and will sell for \$7 per kilogram. The second mix will contain 3/4 nuts and 1/4 raisins, and will sell for \$9.50 per kilogram. How many kilograms of each mix should the company prepare for the maximum revenue?" Let x be kilograms of the first mix which are prepared and let y be kilograms of the second mix. Which of the following is the objective function when this problem is modelled as a linear programming problem?

(a) $150x + 90y$

(b) $\frac{1}{2}x + \frac{1}{2}y$

(c) $\frac{3}{4}x + \frac{1}{4}y$

(d) $\frac{5}{4}x + \frac{3}{4}y$

(e) $7x + 9.5y$

(f) None of the Above.

20. Consider the problem: "A candy company has 150 kg of chocolate-covered nuts and 90 kg of chocolate-covered raisins to be sold as two different mixes. The first mix will contain half nuts and half raisins and will sell for \$7 per kilogram. The second mix will contain 3/4 nuts and 1/4 raisins, and will sell for \$9.50 per kilogram. How many kilograms of each mix should the company prepare for the maximum revenue?" Let x be kilograms of the first mix which are prepared and let y be kilograms of the second mix. Which of the following is a constraint when this problem is modelled as a linear programming problem?

N: 150
R: 90

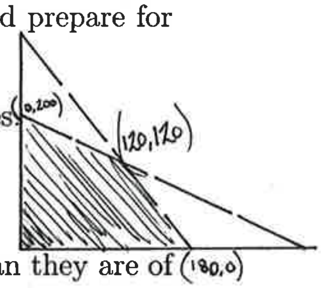
$\frac{1}{2}x + \frac{3}{4}y \le 150$
$\frac{1}{2}x + \frac{1}{4}y \le 90$

$\frac{1}{2}n + \frac{1}{2}r$
$\frac{3}{4}n + \frac{1}{4}r$

(a) $150x + 90y \le 240$
(b) $\frac{1}{2}x + \frac{1}{2}y \le 90$
(c) $\frac{1}{2}x + \frac{3}{4}y \le 150$
(d) $\frac{1}{4}x + \frac{1}{2}y \le 90$
(e) $\frac{7}{x} + 9.5y \le 150$
(f) None of the Above.

21. A candy company has 150 kg of chocolate-covered nuts and 90 kg of chocolate-covered raisins to be sold as two different mixes. One mix will contain half nuts and half raisins and will sell for \$7 per kilogram. The other will contain 3/4 nuts and 1/4 raisins, and will sell for \$9.50 per kilogram. Find how many kilograms of each mix the company should prepare for the maximum revenue. Which of the following statements is true?

X = 120
Y = 120



(a) At the maximum revenue, they are making the same amount of both mixes.
(b) At the maximum revenue, they only make the second mix.
(c) At the maximum revenue, they only make the first mix.
(d) At the maximum revenue, they are making 40 kg more of the first mix than they are of the second.
(e) At the maximum revenue, they are making 40 kg more of the second mix than they are making of the first.
(f) None of the Above.

A.11 Know the terminology associated with using the simplex method to solve linear programming problems: simplex method, standard maximum form, standard minimum form, slack variable, simplex tableau, indicators, basic variables, basic feasible solutions, pivot. (7.1-7.3)

A.12 Be able to identify a linear programming problem in standard maximum form (p.311). Know how to introduce slack variables to create a simplex tableau and find basic variables. Know how to pivot to produce a new tableau. Know how to read basic feasible solutions from a simplex tableau. (7.1)

A.13 Be able to solve standard maximization problems using the simplex method (p.321). In particular, know how to determine where to pivot, and know how to pivot and know how to identify when a solution has been reached. (7.2)

A.14 Be able to identify a linear programming problem in standard minimum form (p.329). Understand the concept of dual systems. Be able to use the 'Theorem of Duality' (p.333) and the method of duals (p.334) to solve a standard minimum problem with duals.

22. Consider the linear programming problems in #18 and #21 on this review. Are they in standard maximum form, standard minimum form, or neither. How do you know?

Problems 23-29 refer to the simplex tableau $\left[\begin{array}{cccccc|c} 1 & 5 & 0 & 1 & 2 & 0 & 6 \\ 0 & 2 & 1 & 2 & 3 & 0 & 15 \\ \hline 0 & 4 & 0 & 1 & -2 & 1 & 64 \end{array}\right]$

23. How many slack variables does this linear programming problem have?

2 (B/C two constraints)

24. Which of the variables in this simplex tableau are basic?

x_1 and x_3

25. What is the basic feasible solution corresponding to this tableau?

(a) $x_1 = 6, x_2 = 0, x_3 = 15, s_1 = 0, s_2 = 0, z = 64$

(b) $x_1 = 0, x_2 = 4, x_3 = 0, s_1 = 1, s_2 = -2, s_3 = 1, z = 64$

(c) $x_1 = 6, x_2 = 15, x_3 = 64$

(d) $x_1 = 1, x_2 = 5, x_3 = 0, s_1 = 1, s_2 = 2, z = 6$

(e) There is no feasible solution for this tableau because it has a negative indicator.

(f) None of the Above.

26. Where is the pivot column for this tableau?

(a) Column 2

(b) Column 4

(c) Column 5

(d) Column 6

(e) Pivoting is NOT the correct action for this tableau.

(f) None of the Above.

27. Which element of this tableau should we pivot on?

(a) (2,2)

(b) (2,4)

(c) (1,5)

(d) (3,6)

(e) Pivoting is NOT the correct action for this tableau.

(f) None of the Above.

28. Pivot on the (1,5) element. How much does the value of z, the objective function, increase or decrease?

$R_1 + R_3 \to R_3$

(a) It increases by 12.

(b) It increases by 6.

(c) It increased by 4.

(d) It decreases by 8.

(e) It decreases by 16

(f) None of the Above.

29. (B) A linear programming problem has five variables $x_1, x_2, \ldots x_5$ and 7 constraints. How many slack variables does the problem have?

(a) 5

(b) 6

(c) 7

(d) 8

(e) 12

(f) None of the Above.

30. A linear programming problem has five variables $x_1, x_2, \ldots, x_5$ and 7 constraints. How many row are in the simplex tableau?

(a) 5
(b) 6
(c) 7
(d) 8
(e) 12
(f) None of the Above..

31. An investor is considering three types of investments: a high-risk venture into oil leases with a potential return of 15%, a medium-risk investment in stocks with a 9% return, and a relatively safe bond investment with a 5% return. He has \$50,000 to invest. Because of the risk, he wants his investment in oil and stocks together to be less than his investment in bonds. Also, his investment in oil leases will be no more than half his investment in stocks. He wants to maximize his returns. Set up an initial tableau for this linear programming problem.

$$\left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 50000 \\ 1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline -.15 & -.09 & -.05 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

32. Which is the correct action for the tableau

$2x_1 \geq x_2$
$2x_1 - x_2 \geq 0$

$$\left[\begin{array}{ccccc|c} -1 & 1 & 0 & 1 & 0 & 1 \\ -4 & 0 & 1 & -2 & 0 & 3 \\ \hline -1 & 0 & 0 & 2 & 1 & 4 \end{array}\right]?$$

(a) The pivot on the $(2, 1)$ entry.
(b) The pivot on the $(2, 2)$ entry.
(c) The pivot on the $(3, 2)$ entry.
(d) There is nothing to be done to this linear programming system, because the objective function is maximized.
(e) There is no maximum for this linear programming problem, because the variable in the pivot column is not bounded. (negatives)
(f) None of the Above.

33. If you are solving the linear programming problem with initial tableau $\left[\begin{array}{cccccc|c} 4 & 5 & 2 & 1 & 0 & 0 & 18 \\ 2 & 8 & 6 & 0 & 1 & 0 & 24 \\ \hline -5 & -3 & -6 & 0 & 0 & 1 & 0 \end{array}\right]$,

after you pivot once, what are the indicators?

(a) $\left[\begin{array}{cccccc|c} -3 & 5 & 0 & 0 & 1 & 1 & 24 \end{array}\right]$
(b) $\left[\begin{array}{ccccc} -3 & 5 & 0 & 0 & 1 \end{array}\right]$
(c) x_3, s_1
(d) x_3, s_1, z
(e) $\begin{bmatrix} 30 \\ 24 \\ 24 \end{bmatrix}$.
(f) None of the Above.

$R_2 + R_3$ $\quad -3 \quad 5 \quad 0 \quad 0 \quad 1 \quad 1 \quad 24$

INDICATORS = VARIABLES, SLACK

34. A linear programming problem in standard maximum form has initial tableau

$$\begin{array}{l} 3R_1-R_2 \\ \\ R_2+R_3 \end{array}\left[\begin{array}{cccccc|c} 4 & 5 & 2 & 1 & 0 & 0 & 18 \\ 2 & 8 & 6 & 0 & 1 & 0 & 24 \\ \hline -5 & -3 & -6 & 0 & 0 & 1 & 0 \end{array}\right].\left[\begin{array}{cccccc|c} 10 & 7 & 0 & 3 & -1 & 0 & 30 \\ 2 & 8 & 6 & 0 & 1 & 0 & 24 \\ \hline -3 & 5 & 0 & 0 & 1 & 1 & 24 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 10 & 7 & 0 & 3 & -1 & 0 & 30 \\ 0 & -33 & -30 & 3 & -6 & 0 & -90 \\ \hline 0 & 71 & 0 & 9 & 7 & 10 & 330 \end{array}\right]$$

Find the maximum for this system.

(a) 10

(b) 24

(c) 33

(d) 240

(e) 330

(f) None of the Above.

35. What is the dual problem to:

Minimize $w = 22y_1 + 44y_2 + 33y_3$ subject to $\left\{\begin{array}{ccccccc} y_1 & + & 2y_2 & + & y_3 & \geq & 6 \\ y_1 & & & + & y_3 & \geq & 3 \\ y_1 & + & 2y_2 & + & 2y_3 & \geq & 8 \\ y_1 & \geq & 0, y_2 & \geq & 0, y_3 & \geq 0 & \end{array}\right\}$

TRANSPOSE

36. In order to minimize $w = 9y_1 + 10y_2$ subject to $\left\{\begin{array}{ccccc} 4y_1 & + & 5y_2 & \geq & 6 \\ 2y_1 & + & 4y_2 & \geq & 7 \\ 3y_1 & + & 1y_2 & \geq & 5 \\ y_1 & \geq & 0, y_2 & \geq & 0 \end{array}\right\}$, several pivots are performed on the the simplex tableau

$$\left[\begin{array}{cccccc|c} 4 & 2 & 3 & 1 & 0 & 0 & 9 \\ 5 & 4 & 1 & 0 & 1 & 0 & 10 \\ \hline -6 & -7 & -5 & 0 & 0 & 1 & 0 \end{array}\right]$$

to get the tableau

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & y_1 & y_2 & z & \end{array}$$
$$\left[\begin{array}{cccccc|c} 3 & 0 & 5 & 2 & -1 & 0 & 8 \\ 11 & 10 & 0 & -1 & 3 & 0 & 21 \\ \hline 47 & 0 & 0 & 13 & 11 & 10 & 227 \end{array}\right] / 10$$

$y_1 = \frac{13}{10}$

$y_2 = \frac{11}{10}$

For what values of y_1 and y_2 is w minimized?

37. Cauchy Canners produces canned corn, beans, and carrots. Demand for vegetables requires production of at least 1000 cases per month. Based on past sales, Cauchy Canners should produce at least twice as many cases of corn as of beans, and at least 340 cases of carrots. It costs \$10 to produce a case of corn, \$15 to produce a case of beans, and \$25 to produce a case of carrots. How many cases of each vegetable should be produced to minimize costs?

x = corn
y = bean
z = carrots

2 corn ≥ bns

340 ≤ CARROTS

$x_1 + x_2 + x_3 \geq 1000$

$340 \leq x_3$

$2x_1 - x_2 \geq 0$

$z = 10c + 15b + 25\text{car}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 0 & 1 & 340 \\ 2 & -1 & 0 & 0 \\ \hline 10 & 15 & 25 & 0 \end{array}\right]^T = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 10 \\ 1 & 1 & -1 & 15 \\ 1 & 0 & 0 & 25 \\ \hline -1000 & -340 & 0 & 0 \end{array}\right]$$

$x = 660$ $z = 340$
$y = 0$

Extension problems: On each exam, roughly 20% of the points will come from what I call extension problems. These are problems that you most likely have not seen before, and which generally require a combination of techniques or a little ingenuity to solve.

39. For the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 4 & 3 & 3 & -1 \\ 3 & 3 & 2 & -2 \\ 1 & 2 & 1 & 0 \end{bmatrix} \left| \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix} \right.$$

Which of the columns below could be the first column of A^{-1}?

(a) $\begin{bmatrix} 1/2 \\ 1/10 \\ -7/10 \\ 1/5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1/4 \\ 1/3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ A^T

(d) $\begin{bmatrix} 0 \\ -2/5 \\ 4/5 \\ 1/5 \end{bmatrix}$

(e) $\begin{bmatrix} 1/2 \\ 3/10 \\ -11/10 \\ -2/5 \end{bmatrix}$

(f) None of the Above.

1:c, 2:e, 3:a, 4:e, 5:b, 6:c, 7:b, 8:a, 9:e, 10:a, 11:e,
12:b, 13:c, 14:e, 15:d, 16:d, 17:c, 18:a, 19:e, 20:c, 21:a,
22:(below), 23:2, 24:(below), 25:a, 26;c, 27:c, 28:b, 29:c, 30:d,
31:(below), 32:e, 33:b, 34:c, 35:(below), 36:y_1=13/10,y_2=11/10,
37: 660 corn, 0 beans, 340 carrots , 38:a

22: #18 neither, #21 standard maximum

24: x_1, x_3

31: $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 50000 \\ 1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -0.15 & -0.09 & -0.05 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

35: Minimize $6x_1 + 3x_2 + 8x_3$ subject to $\left\{ \begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & \leq & 22 \\ 2x_1 & & & + & 2x_3 & \leq & 44 \\ x_1 & + & x_2 & + & 2x_3 & \leq & 33 \\ x_1 & \geq & 0, x_2 & \geq & 0, \quad x_3 & \geq & 0 \end{array} \right\}$