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Math 112
Exam 2

Name:__________________________

Section:_________________________

Instructor:_______________________

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Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
III) Multiple choice questions are 4 points each.
IV) For questions which require a written answer, show all your work in the space provided and justify your answer.
V) Simplify your answers.
VI) Scientific calculators are allowed.
VII) No books or notes are allowed.
VIII) There is no time limit on this exam.

Part I: Multiple Choice Questions: *Mark the correct answer. (4 points each)*

1 What is the maximum value of $f(x) = x^3 - (9/2)x^2 - 12x + 7$ on the interval $[-2, 6]$?

- ☐ 27/2
- ☐ -11
- ☐ -1
- ☐ -17/2
- ☐ 49
- ☐ 5
- ☐ -49
- ☐ 6

2 If $f(x) = \sin(\sin(e^x))$, find $f'(x)$.

- ☐ $e^x \cos(e^x) \cos(\sin(e^x))$
- ☐ $\cos(\sin(e^x))$
- ☐ $2e^x \cos(e^x) \sin(e^x)$
- ☐ $\cos(\cos(e^x))$
- ☐ $e^x \cos(\cos(e^x))$
- ☐ $2 \sin(e^x)$

3 If f, g, and h are differentiable functions, find $(f^2gh)'$.

- ☐ $2ff'gh + f^2g'h + f^2gh'$
- ☐ $2ff' + g' + h'$
- ☐ $2ff'g + f^2g + gh'$
- ☐ $2fghf'g'h'$
- ☐ $2ff'g'h'$
- ☐ $f'gh + fg'h + fgh'$

4 Suppose f is continuous on $[0, 10]$ and differentiable on $(0, 10)$, and assume

$$f(0) = f(5) = 7 \text{ and } f(10) = 12.$$

For which values of a can you be certain that $f'(c) = a$ for some c in $(0, 10)$?

(i) 0

(ii) 1/2

(iii) 1

- ☐ i and iii only
- ☐ all of them
- ☐ ii only
- ☐ i and ii only
- ☐ iii only
- ☐ ii and iii only
- ☐ i only
- ☐ none of them

5 Let $F(x) = f(g(x))$, where f and g are differentiable functions. If $g(2) = 8$, $g'(2) = 2$, $f(2) = 7$, $f'(2) = 3$, $f(8) = 10$, and $f'(8) = 5$, find $F'(2)$.

- ☐ 5
- ☐ 3
- ☐ 6
- ☐ 24
- ☐ 10
- ☐ 7

6 Find $\dfrac{d}{d\theta}\sin(\theta)\tan(\theta)$.

- ☐ $\tan(\theta)(1+\cos(\theta))$
- ☐ $\tan^2(\theta)(1+\cos^2(\theta))$
- ☐ $\sin(\theta)(1+\sec^2(\theta))$
- ☐ $\tan(\theta)(1+\cos^2(\theta))$
- ☐ $\sin(\theta)(1-\sec^2(\theta))$
- ☐ $\sin(\theta)(\sin(\theta)+\sec^2(\theta))$

7 If $f(x) = 5 - x^2$, what values of c in the interval $(0, 1)$ satisfy the conclusion of the Mean Value Theorem?

- ☐ the Mean Value Theorem does not apply
- ☐ 0 only
- ☐ -1/2 only
- ☐ 1/4 only
- ☐ 1/2 and 1/4
- ☐ 1/2 only

8 Find the slope of the tangent line to $x^2 + (3/2)y^2 = 2x^2 + 2y^2 - x$ passing through the point $(2, 2)$.

- ☐ -3/2
- ☐ 0
- ☐ 6
- ☐ 3/2
- ☐ 2
- ☐ 10

9 If $f(x) = \dfrac{5x^4 - 3x}{e^x}$, find $f'(0)$.

- ☐ -3
- ☐ DNE
- ☐ $\dfrac{5}{4}$
- ☐ 0
- ☐ ∞

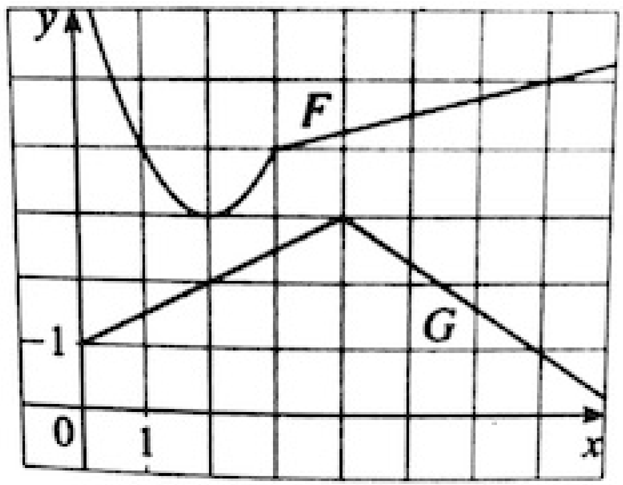
10 The length of a rectangle increases at a rate of 4 inches per second, and the width increases at a rate of 7 inches per second. How fast is the area of the rectangle changing when the length is 10 inches and the width is 20 inches?

- ☐ 180 in^2/sec
- ☐ 150 in^2/sec
- ☐ none of these
- ☐ 200 in^2/sec
- ☐ 28 in^2/sec

11 If $f(x) = x^{2x}$, find $f'(x)$.

- ☐ $2x^{2x}$
- ☐ $(2x)x^{2x-1}$
- ☐ $x^{2x}\ln(x)$
- ☐ $x^{2x} + x^{2x-1}$
- ☐ $2 + 2\ln(x)$
- ☐ $x^{2x}(2 + 2\ln(x))$

12 Let $f(x) = \dfrac{F(x)}{G(x)}$, where $F(x)$ and $G(x)$ are the functions shown below.



Find $f'(7)$.

- ☐ 43/12
- ☐ −37/12
- ☐ 5
- ☐ −3/8
- ☐ 0
- ☐ 1/4

Part II: *Justify your answer and show all work for full credit.*

13 ☐ **0** ☐ **1** ☐ **2** ☐ **3** ☐ **4** ☐ **5** ☐ **6** ☐ **7** *Administrative Use Only*

Use implicit differentiation to show that

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}.$$

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 *Administrative Use Only*

Write the equations of all lines that are tangent to the function $y = \dfrac{2x+3}{x+1}$ and parallel to the line $x + y = 0$.

15 | □0 □1 □2 □3 □4 □5 □6 □7 □8 *Administrative Use Only*

Find $\dfrac{dy}{dx}$ for each of the following:

a. $x^2 + xe^y + y^2 = \ln(y^2)$

b. $y = x^{\sin x}$

16 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** *Administrative Use Only*

Show that $x^3 + 12x + 5 = 0$ has exactly one solution on the interval $[-2, 2]$. Name any theorems you use.

17 □0 □1 □2 □3 □4 □5 □6 □7 *Administrative Use Only*

[*Note: For the following problem, recall that the volume of a sphere is given by*

$$V(r) = \frac{4}{3}\pi r^3$$

and the surface area is given by

$$S(r) = 4\pi r^2.$$

You may leave your answers in terms of π, *but be sure to include units on your answers.*]

A spherical snowball melts so that its **surface area** decreases at a rate of 1 cm^2/min.

a. Find the rate at which the **radius** of the snowball is decreasing when the volume is $36\pi\ \text{cm}^3$.

b. Find the rate at which the **volume** of the snowball is decreasing when the volume is $36\pi\ \text{cm}^3$.

18 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** *Administrative Use Only*

Given the function $y = [\ln(\sqrt{x} - 4)]^2$,

a. Find $\dfrac{dy}{dx}$ for the given function.

b. At what point on the graph of the given function is the tangent line horizontal?

19 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** $\square$ **8** *Administrative Use Only*

Given $f(x) = x^3 + 6x^2 - 15x$,

(a) Find the slope of the secant line over the interval $[0, 5]$.

(b) Is there a tangent line on the interval $(0, 5)$ that is parallel to the secant line described in part (a)? How do you know? [*Hint: you do not need to find the tangent line.*]

(c) Find the absolute maximum and absolute minimum values of f on $[0, 5]$.