Math 112  
Exam 2  
March 7-?, 2016  
(Late Day: March ?, 2016)

Encode your BYU ID in the grid below.

Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.

II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 5 points each.

III) For questions which require a written answer, show all your work in the space provided and justify your answer.

IV) Simplify your answers.

V) No books, notes, or calculators of any type are allowed.

VI) There is no time limit on this exam.
Part I: Multiple Choice Questions: Mark the correct answer. (4 points each)

Use the following data for questions 1–3.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>f(x)</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>-3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>f'(x)</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>-2</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>g</td>
<td>g(x)</td>
<td>3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>g'(x)</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Let \( h_1(x) = f(x)g(x) \), \( h_2(x) = \frac{f(x)}{g(x)} \), and \( h_3(x) = f(g(x)) \).

1. Find \( h_1'(5) \).
   - [ ] 10
   - [ ] 12
   - [ ] 15
   - [ ] 11
   - [ ] 9

2. Find \( h_2'(0) \).
   - [ ] 7/9
   - [ ] -1/2
   - [ ] 2
   - [ ] -1
   - [ ] 1/3
3. Find $h'_3(4)$.

- $-1$
- $9$
- $-3$
- $0$
- $2$

4. Find the value of $c$ that satisfies the conclusion of the mean value theorem for the function $x^3 - 3x$ on the interval $[0, 2]$.

- $1$
- $2/\sqrt{3}$
- $\sqrt{5}/3$
- $2$
- $\sqrt{1/3}$

5. Evaluate $\lim_{\theta \to 0} \frac{\sin(3\theta) \sin(5\theta)}{2\theta^2}$.

- $15/2$
- $4/15$
- $2/15$
- $\infty$
- $15/4$
- $0$
6 If the total cost (in dollars) to produce \( x \) widgets is given by \( C(x) = \frac{x^2}{100} + 2x + 10 \), find an equation for the marginal cost of the \( x \)th widget.

- \( \frac{3x^2}{100} + 4x \)
- \( \frac{x^2}{50} + 2x \)
- \( \frac{x}{50} + 2 \)
- \( \frac{1}{100} - \frac{10}{x^2} \)
- \( \frac{x}{100} + 2 + \frac{10}{x} \)

7 The population (in millions of cells) of a certain bacteria culture is given by \( f(t) = 3^t \), where \( t \) is measured in hours. Over which interval of time is the average growth rate the largest?

- \([0, 3]\)
- \([1, 2]\)
- \([1, 3]\)
- \([0, 1]\)
- \([0, 2]\)
- \([2, 3]\)
8 Suppose the maximum value for $f(x)$ in the interval $[a, b]$ occurs at the point $c$. Which of the following CANNOT be true:

- $c = a$
- $c = b$
- $f'(c) = 0$
- $f'(c)$ does not exist
- All of the above are possible

9 On Planet Math, bubbles are cubes instead of spheres. Suppose a cube-shaped bubble is being blown. If the volume of the cube is steadily increasing at $36 \text{ cm}^3/\text{s}$, at what rate are the sides of the cube increasing when the volume is $27 \text{ cm}^3$?

- $4 \text{ cm/s}$
- $3 \text{ cm/s}$
- $4/3 \text{ cm/s}$
- $2 \text{ cm/s}$
- $\sqrt{36} \text{ cm/s}$

10 For which equations does Rolle’s Theorem apply on the interval $[0, 1]$?

(i) $x^4 - x^2$  (ii) $\sin(\pi x)$  (iii) $e^x$

- (i) only
- (i) and (ii)
- (ii) only
- (i), (ii), and (iii)
- (iii) only
The following three graphs represent the position, velocity, and acceleration of
an object. Match each graph with the function.

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\[
\begin{array}{ccc}
\text{1} & \text{2} & \text{3} \\
\end{array}
\]

- [ ] 1 is position, 2 is velocity, and 3 is acceleration
- [ ] 2 is position, 1 is velocity, and 3 is acceleration
- [ ] 3 is position, 1 is velocity, and 2 is acceleration
- [ ] 1 is position, 3 is velocity, and 2 is acceleration
- [ ] 3 is position, 2 is velocity, and 1 is acceleration
- [ ] 2 is position, 3 is velocity, and 1 is acceleration
Part II: Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

Evaluate the given derivative:

(a) \( \frac{d}{dx} \csc(2x) \)

(b) \( \frac{d}{dx} \sin^{-1}(x) \)

(c) \( \frac{d}{dx} \ln \left( \sqrt{\frac{x}{x+1}} \right) \)

(d) \( \frac{d}{dx} 2^{\ln(x)} \)

(e) \( \frac{d^{101}}{dx^{101}} \sin(x) + e^x + x^6 \)
Find $y'$ if:

(a) $y = (\sin(x))^{\tan(x)}$

(b) $x^3 + 3x^2y + y^3 = 12$

(c) $y = e^{\cos^2(2\pi + 1)} - x^2$
Find the equation for the line tangent to $x \sin(y) = y \sin(x)$ at the point $(\pi, \pi)$. 
The position (in feet) of an object dropped of a 160-foot tall cliff is given by \( s(t) = 160 - 16t^2 \), where \( t \) is measured in seconds.

(a) Find the time at which the object hits the ground.

(b) Find the velocity of the object when it hits the ground.
Find the maximum and minimum values of \( \frac{x^4}{4} - 2x^3 + 4x^2 + 2 \) on the interval \([-2, 3]\).
A boat is being pulled into a dock by a rope by a winch. If the winch is five feet above the level of the water and the rope is being pulled in a rate of 2 feet per second, find the rate at which the boat is approaching the dock at the moment when the length of the rope is 13 feet.
Show, using implicit differentiation, that the derivative of \( \tan^{-1}(x) \) is \( \frac{1}{1 + x^2} \).
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