+1/1/60+

NS

40104780

001

Math 113
Exam 3

Nov 18, 2016
(Late Day: Nov 22, 2016)

Name:_____

Section:_____

Instructor:_____

Encode your BYU ID in the grid below.

☐0	☐0	☐0	☐0	☐0	☐0	☐0	☐0	☐0
☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1
☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2
☐3	☐3	☐3	☐3	☐3	☐3	☐3	☐3	☐3
☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4
☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5
☐6	☐6	☐6	☐6	☐6	☐6	☐6	☐6	☐6
☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7
☐8	☐8	☐8	☐8	☐8	☐8	☐8	☐8	☐8
☐9	☐9	☐9	☐9	☐9	☐9	☐9	☐9	☐9

Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 5 points each.
III) For questions which require a written answer, show all your work in the space provided and justify your answer.
IV) Simplify your answers.
V) No books, notes, or calculators of any type are allowed.
VI) There is no time limit on this exam.

Part I: Multiple Choice Questions: *Mark the correct answer. (5 points each)*

1 Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{nx^{2n-1}}{4^n}$

- ☐ 4
- ☐ $\frac{1}{4}$
- ☐ 1
- ☐ ∞
- ☐ 3
- ☐ $\frac{1}{2}$
- ☐ 2
- ☐ 0

2 Determine which **ONE** of the following series is conditionally convergent (i.e. convergent but not absolutely convergent).

- ☐ $\sum_{n=2}^{\infty} \frac{(-1)^{2n}}{n+1}$
- ☐ $\sum_{n=3}^{\infty} \frac{(-1)^n}{n^2-n}$
- ☐ $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$
- ☐ $\sum_{n=1}^{\infty} \frac{\sin(n)}{3^n}$
- ☐ $\sum_{n=1}^{\infty} \frac{n(-1)^n}{n^2+5n+1}$

3 Find all the values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

is convergent.

- ☐ $p \geq 1$
- ☐ Diverges for all p
- ☐ $p \geq 2$
- ☐ $p > 1$
- ☐ $p > 2$
- ☐ $p > e$
- ☐ $0 < p \leq 1$
- ☐ $0 < p < 1$

4 For the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n)!}$ find the smallest n such that the alternating series estimation theorem ensures a remainder less than .001.

- ☐ 3
- ☐ 1
- ☐ 5
- ☐ None, since the series diverges.
- ☐ 2
- ☐ 4
- ☐ 6

5 Let $\sum_{n=0}^{\infty} c_n(x-3)^n$ be a power series. Suppose we know that the series converges at $x = 6$ and divergent when $x = -2$. Then at which of the following points do we know the series converges?

- ☐ x=7
- ☐ x=9
- ☐ x=0
- ☐ x=1
- ☐ x=-1

6 Which of the following is a **false** statement?

- ☐ If $\sum a_n$ and $\sum b_n$ have positive terms and $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, then either both series converge or both diverge.
- ☐ Let $\sum a_n$ and $\sum b_n$ have positive terms. If $\sum a_n$ is divergent and $a_n \leq b_n$ for all n, then $\sum b_n$ is divergent.
- ☐ Let $\sum a_n$ and $\sum b_n$ have positive terms. If $\sum b_n$ is divergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is convergent.
- ☐ If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- ☐ If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L > 1$, then the sequence $\{a_n\}$ is divergent.

7 Exactly one of the following **sequences** diverges. Determine which one. In each case, the n-th term of the sequence is given.

- ☐ $a_n = \dfrac{5n^3 + 2}{n^3 - n}$
- ☐ $a_n = \dfrac{4^n}{1 + 7^n}$
- ☐ $a_n = \cos\left(\dfrac{n\pi}{n+3}\right)$
- ☐ $a_n = \dfrac{\ln n}{\ln 2n}$
- ☐ $a_n = \dfrac{\tan^{-1} n}{n}$
- ☐ $a_n = \dfrac{2n^2 - n}{\sqrt{n^3 + 3n^2}}$

8 Find a power series representation for $f(x) = \ln(1 - 3x)$.

- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{3^{n-1}(n+1)}$
- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{(-3)^{n}(n+1)}$
- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{3(n+1)}$
- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{-(3^{n+1})x^{n+1}}{n+1}$
- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^{n}x^{n+1}}{n+1}$
- ☐ $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n}x^{n+1}}{n+1}$

Part II: *Justify your answer and show all work for full credit.*

9 ☐**0** ☐**1** ☐**2** ☐**3** ☐**4** ☐**5** ☐**6** ☐**7** ☐**8** ☐**9** ☐**10 DON'T MARK**

Below are two geometric series. Determine for each one, whether it **converges or diverges**, and, if convergent, **calculate the sum**.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{4^{2n}}{6^{n-1}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{5 \cdot 2^{n-1}}{3^n}$

10 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

Estimate an upper bound of the error for the sum of the first 100 terms for the series

$$\sum_{n=1}^{\infty} \frac{3}{5+n^5}.$$

11 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

Below are series. Determine for each one, whether it **converges or diverges**. State the test you use and show your work.

1. $\sum_{n=1}^{\infty} \frac{2^n}{3^n n^3}$

2. $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+3}\right)^{5n}$

12 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

Below are series. Determine for each one, whether it **converges or diverges**. State the test you use and show your work.

1. $\sum_{n=1}^{\infty} ne^{-n^2}$

2. $\sum_{n=2}^{\infty} \frac{n+1}{n^3-1}$

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

Find the **interval** of convergence of the power series $\sum_{n=2}^{\infty} \frac{(x-2)^n}{\sqrt{n}-1}$.

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

Find a power series representation for

$$f(x) = \frac{1}{(2x-1)^2}$$

and find the **radius** of convergence for the series.