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Math 112
Exam 3
Nov 29 - Dec 02, 2016

Name:_____

Section:_____

Instructor:_____

Encode your BYU ID in the grid below.

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Instructions

1. Do not write on the barcode area at the top of each page, or near the four circles on each page.
2. Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
3. For questions which require a written answer, show all your work in the space provided and justify your answer.
4. Simplify your answers.
5. No books or notes are allowed.
6. There is no time limit on this exam.

Part I: Multiple Choice Questions: *(4 points each) Choose the best answer for each multiple choice question. Fill in the box completely for the the correct answer.*

1 Find $\lim_{t\to 0} \dfrac{\cos t - 1}{t^2}$.

- ☐ -24
- ☐ ∞
- ☐ 1
- ☐ $\dfrac{1}{2}$
- ☐ $-\dfrac{1}{2}$
- ☐ 0
- ☐ 24
- ☐ $-\dfrac{1}{24}$

2 Given that

$$\lim_{x\to a} f(x) = 0 \qquad \lim_{x\to a} g(x) = +\infty, \tag{1}$$

which of the following limits are indeterminate forms?

A $\lim_{x\to a} \dfrac{f(x)}{g(x)}$

B $\lim_{x\to a} \dfrac{g(x)}{f(x)}$

C $\lim_{x\to a} f(x)g(x)$

D $\lim_{x\to a} \left[\dfrac{1}{g(x)}\right]^{f(x)}$

- ☐ B only
- ☐ A and D
- ☐ C and D
- ☐ B, C, and D
- ☐ all of them
- ☐ C only
- ☐ A and B

3 Find the interval(s) on which $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing.

- ☐ $(1, 2)$
- ☐ $(-3, 4) \cup (5, 6)$
- ☐ $(0, 1)$
- ☐ $(-\infty, 1)$
- ☐ $(-2, 2)$
- ☐ $(-\infty, 1) \cup (2, +\infty)$
- ☐ $(2, +\infty)$
- ☐ $(-\infty, -2) \cup (2, +\infty)$

4 Find the linearization of e^x at $x = 0$.

- ☐ $y = e^{-x}$
- ☐ $y = 1 + x$
- ☐ $y = 2 + x$
- ☐ $y = 2 - x$
- ☐ $y = e^x$
- ☐ $y = x$
- ☐ $y = 1 - xe^x$
- ☐ $y = 2x + 2$

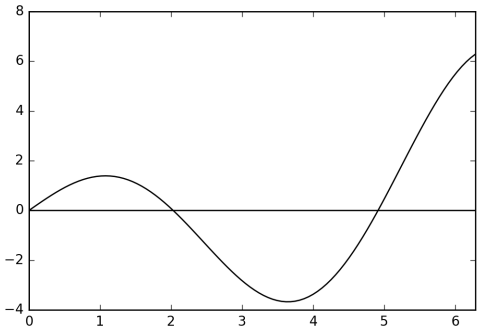
5 Using Newton's method to approximate the zero of $f(x) = 3x^2 - 3x + 1$, determine the second approximation if the initial one is $x_1 = 2$.

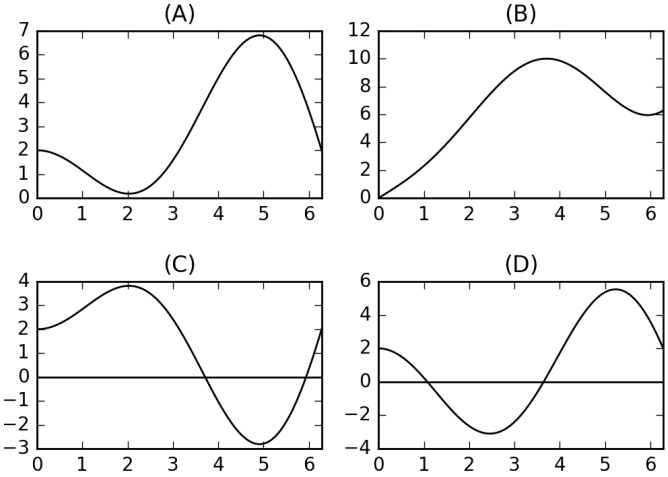
- ☐ -1
- ☐ $\frac{11}{9}$
- ☐ 0
- ☐ 2
- ☐ $\frac{1}{6}$
- ☐ $\frac{7}{9}$
- ☐ 9
- ☐ $-\frac{13}{9}$

6 If $\int_1^2 f(x)dx = 3$, $\int_3^2 f(x)dx = -4$, and $\int_1^3 g(x)dx = 6$, then $\int_1^3 [2f(x) - g(x)]\,dx$ is:

- ☐ 1
- ☐ 2
- ☐ 8
- ☐ 4
- ☐ -5
- ☐ -2
- ☐ -3
- ☐ 0

7 The graph of $f'(x)$ is shown first, which of the following options shows the graph of $f(x)$ given that $f(0) = 2$?





- [ ] A
- [ ] D
- [ ] C
- [ ] B

8 On what interval(s) is $f(x) = 3x^4 - 4x^3 + 2$ concave up?

- ☐ $(-\infty, 2/3)$
- ☐ $(-1, 2) \cup (3, +\infty)$
- ☐ $(-\infty, -2/3) \cup (2/3, \infty)$
- ☐ $(-\infty, 0)$
- ☐ $(-\infty, 0) \cup (2/3, +\infty)$
- ☐ $(0, 2/3)$

9 $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \left[\left(\frac{2i}{n} \right)^2 - \cos\left(\frac{2i}{n} \right) \right]$ is equal to what definite integral?

- ☐ $\int_0^1 x^2 dx$
- ☐ $\int_0^2 x^2 dx$
- ☐ $\int_0^2 (x - \cos x)\, dx$
- ☐ $\int_0^2 \left(x^2 - \cos x\right) dx$
- ☐ $\int_0^4 \cos x dx$
- ☐ $\int_0^1 (x \cos x)\, dx$
- ☐ $\int_0^2 \cos x dx$
- ☐ $\int_0^1 \left(x^2 - \cos x\right) dx$

10 Find the point on the line $y = 3x + \frac{3}{2}$ that is closest to the point $(2, 0)$.

- ☐ $\left(-\frac{1}{2}, 0\right)$
- ☐ $\left(-1, -\frac{3}{2}\right)$
- ☐ $\left(-\frac{1}{4}, \frac{3}{4}\right)$
- ☐ $\left(\frac{1}{2}, 3\right)$
- ☐ $\left(\frac{1}{4}, \frac{9}{4}\right)$
- ☐ $\left(1, \frac{9}{2}\right)$
- ☐ $\left(0, \frac{3}{2}\right)$

11

time (second)	0	2	4	6
velocity (meters/second)	40	30	15	0

There is an obstruction on a train track 200 meters ahead so the engineer engages the brakes. The table above gives the velocity of the train at a few select seconds after the brake is engaged. Which of the following are true?

- ☐ There isn't enough information to tell if the train hits the obstruction.
- ☐ The train hits the obstruction between 4 and 6 seconds.
- ☐ The train stops before hitting the obstruction.
- ☐ The train hits the obstruction between 2 and 4 seconds.
- ☐ The train hits the obstruction between 0 and 2 seconds.

12 Write the following sum in sigma notation:

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}$$

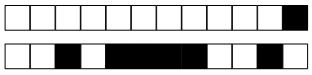
- ☐ $\sum_{k=1}^{6} \frac{1}{k}$
- ☐ $\sum_{k=0}^{6} \frac{1}{k}$
- ☐ $\sum_{k=1}^{6} k^2$
- ☐ $\sum_{k=0}^{6} k^2$
- ☐ $\sum_{k=1}^{6} \frac{1}{k^2}$
- ☐ $\sum_{k=0}^{6} \frac{1}{k^2}$

13 What is the value of $\sum_{i=1}^{n}(2 - 5i)$?

- ☐ $\frac{2}{5}n(1-n)$.
- ☐ $-\frac{5}{2}n^2$.
- ☐ n^2.
- ☐ $\frac{5n(n+1)}{2}$.
- ☐ $2n$.
- ☐ $-\frac{5n^2+n}{2}$.

14 What is the general antiderivative of $g(t) = \dfrac{1}{t} - e^t + t^4$?

- ☐ $\ln t - e^t + \dfrac{1}{4}t^5 + C$
- ☐ $\dfrac{1}{t^2} - e^t + t^4 + C$
- ☐ $\ln t - e^t + 4t^3 + C$
- ☐ $\ln t - e^t + \dfrac{1}{5}t^5 + C$
- ☐ $-\dfrac{1}{t} + e^t - \dfrac{1}{4}t^5 + C$
- ☐ $-\dfrac{1}{t^2} - e^t + \dfrac{1}{5}t^5 + C$



Part II: Free Response Questions: *Neatly write solutions for these problems directly on the exam paper. (Work on scratch paper will not be graded.)*

15 ☐**0** ☐**1** ☐**2** ☐**3** ☐**4** ☐**5** ☐**6** ☐**7 DON'T MARK**

Find the following limit:

$$\lim_{x\to\infty} x^{e^{-x}}.$$

16 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 **DON'T MARK**

A Find the general antiderivative of $f(x) = \dfrac{3x^3 - x + 4}{x^2}$.

B Identify the function $f(x)$ that satisfies:

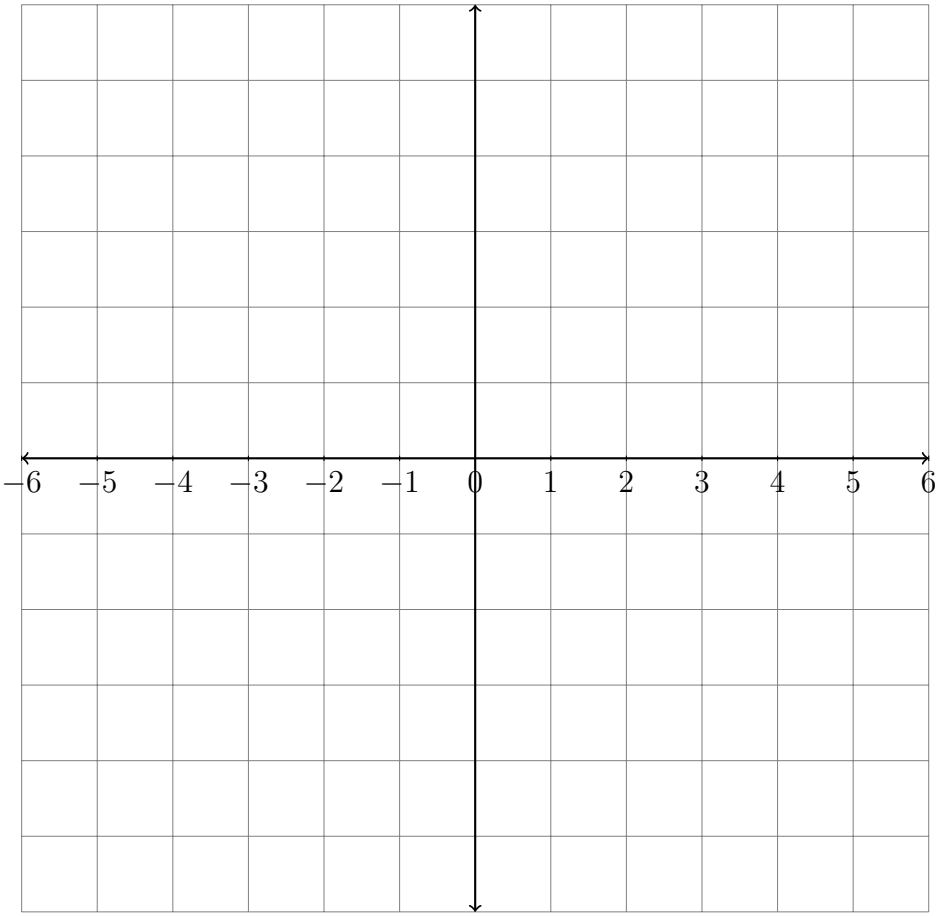
$$f''(x) = -\sin x + e^x, \quad f(0) = -1, \quad f'(0) = 3.$$

17 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 **DON'T MARK**

Sketch the graph of a function that satisfies the following conditions:

- $f(x)$ is continuous and differentiable on $(-\infty, \infty)$.
- $f(0) = 0$.
- $f'(-3) = f'(2) = f'(4) = 0$.
- $f'(x) < 0$ on $(-\infty, -3) \cup (2, 4)$.
- $f'(x) > 0$ on $(-3, 2) \cup (4, +\infty)$.
- $f(x)$ is concave up on $(-\infty, -1)$ and $(3, +\infty)$.
- $f(x)$ is concave down on $(-1, 3)$.

(**You may do your work on the other side. Sketch the graph below.**)



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18 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 **DON'T MARK**

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-0.6	0.3	0.9	1.4	1.8

The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)dx$ using three equal subintervals with (A) right endpoints, (B) left endpoints, and (C) midpoints.

A right endpoints.

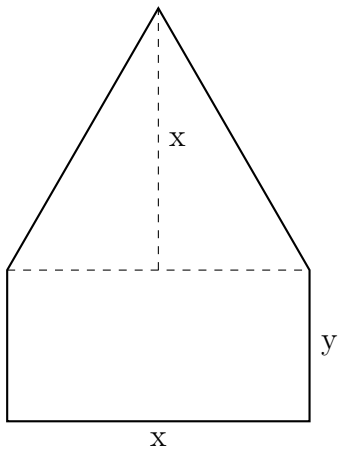
B left endpoints.

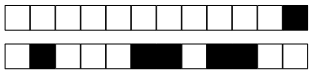
C midpoints.

D If the function is known to be an increasing function, can you say whether your estimates above are less than or greater than the exact value of the integral?

19 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 **DON'T MARK**

A window is formed by placing an isoceles triangle on top of a rectangle as shown in the figure below. If the total perimeter of the window needs to be 4 meters, then what is the maximum area that the window can have? What are the dimensions of this window?





20 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 **DON'T MARK**

Find the limit: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 1\right]$.