Math 113
Exam 2
Mar 1-3, Late Day Mar 4,
2016

Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.

II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 4 points each.

III) For questions which require a written answer, show all your work in the space provided and justify your answer.

IV) Simplify your answers.

V) No books, notes, or calculators of any type are allowed.

VI) There is no time limit on this exam.
FERPA Permission: Please indicate whether you give permission for your exam to be returned to you by email. This question supersedes any permission you have given previously. Please answer it correctly. No score will be assigned to this question. Note: If you choose not to give permission, you will need to discuss with your instructor how you will get your exam.

☐ No, I do not give permission.
☐ Yes, I give permission.

Part I: Multiple Choice Questions: Mark the correct answer. (4 points each)

1 Suppose \( f(x) \) is given by \( \frac{x^4}{4} - x^3 \). Identify the correct statement about local extrema (max/min) of \( f(x) \).

☐ \( f(x) \) has a local maximum at \( x = 3 \) and a local minimum at \( x = 0 \)
☐ \( f(x) \) has a local maximum at \( x = 3 \)
☐ \( f(x) \) has a local minimum at \( x = 3 \)
☐ \( f(x) \) has a local minimum at \( x = 3 \) and a local minimum at \( x = 0 \)
☐ \( f(x) \) has a local maximum at \( x = 3 \) and a local maximum at \( x = 0 \)
☐ \( f(x) \) has a local minimum at \( x = 3 \) and a local maximum at \( x = 0 \)

2 Using Newton’s Method to find a root of \( f(x) = x^3 - x - 1 \) with \( x_1 = 2 \), what is the value of \( x_2 \)?

☐ \(-27/11\)
☐ \(27/11\)
☐ \(-1/5\)
☐ \(-21/5\)
☐ \(21/5\)
☐ \(1/5\)
☐ \(17/11\)
☐ \(-17/11\)
3. Estimate the area under \( y = x^2 \) on the interval \([2, 6]\) using 4 equal-width rectangles and right endpoints.

- \( 54 \)
- \( 70 \)
- \( 69 \)
- \( 86 \)
- \( \frac{208}{3} \)

4. Evaluate \( \int_{0}^{3} \sqrt{9 - x^2} + 2x + 1 \, dx \).

- \( \frac{9}{2} \pi + 12 \)
- \( \frac{9}{2} \pi + 21 \)
- \( \frac{9}{4} \pi + 18 \)
- \( \frac{9}{4} \pi + 18 \)
- \( \frac{9}{2} \pi + 21 \)
- \( \frac{9}{4} \pi + 12 \)

5. Evaluate \( \lim_{x \to 0} \frac{x^2}{1 - \cos(x)} \).

- \( \frac{1}{2} \)
- \(-2\)
- \(0\)
- \(2\)
- Does Not Exist
6 Suppose that \( f \) is a continuous and differentiable function. Which of the following conditions would guarantee that \( f \) has a local minimum at \( x = c \)?

- \( f'(c) < 0, f''(c) = 0 \)
- \( f'(c) > 0, f''(c) = 0 \)
- \( f'(c) = 0, f''(c) > 0 \)
- \( f'(c) = 0, f''(c) < 0 \)
- \( f'(c) = 0, f''(c) = 0 \)

7 Which of the following is an antiderivative for \( f(x) = \frac{1}{1 + x^2} \)? (Hint: Remember what it means to be an antiderivative.)

- \( (1 + x^2)^{-1} \)
- \( \tan^{-1}(x) \)
- \( x - \frac{1}{x} \)
- \( \frac{1}{\sqrt{1 + x^2}} \)
- \( \ln(x^2 + 1) \)

8 If \( x \) and \( y \) are two positive numbers with \( x + y = 20 \), what is the maximum value of \( xy \)?

- 200
- 20
- 10
- 400
- 100
Let \( f(x) \) be the function shown below. Let \( L_n \) and \( R_n \) be the left- and right-sided approximations of the area under the curve on the interval \([0, 4]\) using \( n \) equal-width rectangles. Let \( I = \int_{0}^{4} f(x) \, dx \). Which of the following is true?

- \( L_n < R_n < I \)
- \( R_n < I < L_n \)
- \( I < L_n < R_n \)
- \( R_n < L_n < I \)
- \( L_n < I < R_n \)
- \( I < R_n < L_n \)

Which of these is incorrect?

- \( \int_{a}^{b} f(x) \, dx + \int_{c}^{d} f(x) \, dx = \int_{c}^{d} f(x) \, dx \)
- If \( f(x) \geq 0 \) on \([a, b]\), then \( \int_{a}^{b} f(x) \, dx \geq 0 \).
- \( \int_{a}^{b} f(x) + g(x) \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \)
- \( \int_{a}^{b} cf(x) \, dx = c \int_{a}^{b} f(x) \, dx \)
- \( \int_{a}^{b} f(x) \, dx = \int_{b}^{a} f(x) \, dx \)
11 Evaluate \[ \sum_{i=1}^{9} \frac{1}{i^2} - \frac{1}{(i+1)^2}. \]

\[ \begin{array}{c}
80 \\
81 \\
99 \\
100 \\
82 \\
81 \\
101 \\
100 \\
1 \\
\end{array} \]

12 If we wish to use Newton’s method in order to approximate the middle root of the equation in the graph shown below, which would be the best starting estimate?

\[ \begin{array}{c}
x_1 = -0.5 \\
x_1 = 0.5 \\
x_1 = 3.5 \\
x_1 = 2.5 \\
x_1 = 1.5 \\
\end{array} \]
Part II: Justify your answer and show all work for full credit.

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Use linear approximation to estimate the value of $\sqrt{3.9}$. 
For the function $f(x) = \frac{x - 3}{x^2}$,

- Find the interval(s) on which the function is increasing / decreasing
- Find the interval(s) on which the function is concave up / concave down
- Find the equations of all horizontal and vertical asymptotes
Using the grid below, sketch the graph of a function that satisfies the following properties:

- \( f'(x) > 0 \) on \((-\infty, -2) \cup (1, \infty)\)
- \( f'(x) < 0 \) on \((-2, 1)\)
- \( f'(-2) \) is undefined
- \( f''(x) > 0 \) on \((-\infty, -2) \cup (-2, 3)\)
- \( f''(x) < 0 \) on \((3, \infty)\)
- \( \lim_{x \to \infty} f(x) = 1 \)
- \( \lim_{x \to -\infty} f(x) = -2 \)
Evaluate the following limits:

- \( \lim_{x \to \infty} x^{\frac{1}{x}} \)

- \( \lim_{x \to \pi^+} \sec(x) - \tan(x) \)
A rancher needs to fence in two rectangular fields for her sheep and cows. In order to be as economical as possible, the two fields will be adjacent, so one length of fence borders both fields. If the rancher can afford exactly 42 km of fencing, what is the largest total area she can enclose?
Evaluate the integral \[ \int_0^3 (x^2 + x) \] by first expressing the integral as a limit of a Riemann sum, and then evaluating that limit. (No credit will be given for use of the Fundamental Theorem of Calculus.)
A vehicle traveling at 120 ft/sec applies the brakes, and comes to a complete stop six seconds later. If deceleration is constant (e.g. \( a(t) = k \) for some negative number \( k \)), determine the total distance traveled before the vehicle comes to a stop.