Math 112
Exam 1
February 8-10, 2016
(Late Day: February 11, 2016)

Encode your BYU ID in the grid below.

Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.

II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 5 points each.

III) For questions which require a written answer, show all your work in the space provided and justify your answer.

IV) Simplify your answers.

V) No books, notes, or calculators of any type are allowed.

VI) There is no time limit on this exam.
Part I: Multiple Choice Questions: Mark the correct answer. (3 points each)

1. Suppose the movement of an object is described by the equation \( y = 2t - 4t^2 \) where \( y \) indicates its distance from a starting point, in feet, after time, \( t \), in seconds. Find the average velocity of the object over the interval from 2 to 5 seconds.

\[
\frac{f(2) - f(5)}{2 - 5} = \frac{[2(5) - 4(5)^2] - [2(2) - 4(2)^2]}{5 - 2}
\]

\[
\frac{[10 - 4 \cdot 25] - [4 - 4 \cdot 4]}{3} = \frac{-90 + 12}{3} = -24
\]

\[\square\] 25 ft/sec
\[\square\] 16 ft/sec
\[\square\] 24 ft/sec
\[\square\] -26 ft/sec
\[\square\] -34 ft/sec

2. Consider the functions, \( f(x) = \frac{x}{x+1} \) and \( g(x) = \frac{x}{x} \). Select the correct statement from the answer options below:

\[\square\] Both functions are neither even nor odd
\[\square\] \( f(x) \) is even and \( g(x) \) is odd
\[\square\] \( f(x) \) is neither even nor odd and \( g(x) \) is even
\[\square\] \( f(x) \) is odd and \( g(x) \) is even
\[\square\] \( f(x) \) is neither even nor odd and \( g(x) \) is odd

\[
\begin{align*}
f(-x) &= \frac{-x}{-x+1} \quad \text{Neither} \\
g(-x) &= \frac{-x}{-x} = \frac{-x}{x} = -x \quad \text{Odd}
\end{align*}
\]

\[
\begin{align*}
\text{Even:} & \quad f(-x) = f(x) \\
\text{Odd:} & \quad f(-x) = -f(x)
\end{align*}
\]
3 The value of the constant \( k \) that makes \( f(x) = \begin{cases} (x - 2)^2, & \text{if } x \geq 0 \\ -3x + k, & \text{if } x < 0 \end{cases} \) continuous everywhere is:

- \( 2 \)
- \( \frac{2}{3} \)
- \( 0 \)
- \( \frac{3}{2} \)
- \( 4 \)
- \( -2 \)
- \( 2 \)

Plug in 0 & solve for \( k \).

\((0 - 2)^2 = -3(0) + k\)
\((-2)^2 = k\)
\(k = 4\)

4 Given \( f(x) = 2x - 1 \), \( g(x) = x^2 \), and \( h(x) = 1 - x \), find and simplify \( f \circ g \circ h \).

- \(-4x^2 + 4x\)
- \(2x^2 - 4x + 1\)
- \(x^2 - 7x + 2\)
- \(x^2 + 6x + 1\)
- \(2x^2 + 12x + 2\)

\(f \circ g \circ h = f(g(h(x)))\)
\(g(h(x)) = (1-x)^2\)
\(f(g(h(x))) = 2(1-x)^2 - 1\) (plug \( h(0) \) in \( g(x) \))
\(= 2 - 4(1-x) + 2x^2 - 1\)
\(= 2 - 4 + 4x + 2x^2 - 1\)
\(= 2x^2 + 4x + 1\)

5 Find all values of \( x \) for which the function is discontinuous: \( f(x) = \frac{-5 + x}{3x(3x + 1)} \).

- \( x = 0 \) and \( x = -\frac{1}{3} \)
- \( x = 0 \)
- \( x = 0 \) and \( x = \frac{1}{3} \)
- \( x = 5 \)
- \( x = 3 \) and \( x = -3 \)

Can't have 0 in denominator...

\(3x(3x + 1)\)

Solve for which values of \( x \) produce 0's in denominator.

\(3x = 0\)
\(x = 0\)
\(3x + 1 = 0\)
\(3x = -1\)
\(x = -\frac{1}{3}\)
6 The graph of $f(x)$ is shown below left. Find an expression for the graph shown below right.

- $2f(x + 2)$
- $f(x + 2) - 2$
- $2f(x)$
- $f(x - 2) + 2$
- $2f(x - 2)$

**Friendly tip:**
When shifting or reflecting graphs, stretch, reflect first, then shift.

7 Calculate this limit: $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$.

- 8
- $\infty$
- $-\infty$
- 4
- $-4$
- the limit doesn’t exist

\[
\frac{x^2 - 16}{x - 4} = \frac{(x + 4)(x - 4)}{x - 4} = x + 4
\]

or
\[
\lim_{x \to 4} \frac{x + 4}{1} = \lim_{x \to 4} (x + 4) = 8
\]
8. Find the limit: \( \lim_{{x \to 1}} e^{x^2-x} \).

- 0
- 1
- \( e \)
- the limit doesn't exist
- \(-1\)

\[ e^{1^2-1} = e^1 = e \neq 1 \]

Tip: always try plugging in limit first!

9. Find all horizontal and vertical asymptotes of the function \( y = \frac{5 + 4x}{x + 3} \).

- horizontal at \( y = \frac{5}{3} \), vertical at \( x = 4 \)
- horizontal at \( y = 4 \), vertical at \( x = -3 \)
- horizontal at \( y = 3 \), vertical at \( x = 4 \)
- horizontal at \( x = -3 \), no vertical
- no horizontal or vertical asymptotes

\[ \frac{5+4x}{x+3} = \]

\[ y = \frac{5+4x}{x+3} \]

\[ \text{V: } x+3 = 0 \quad (\text{set denominator to } 0) \]

\[ x = -3 \]

\[ (\text{take limit of } f(x)) \]

\[ y = \lim_{{x \to \infty}} \frac{5+4x}{x+3} \]

\[ = \]

\[ y = 1 \]

10. \( \tan(\sin^{-1} x) \) can be simplified to which of the following:

- \( \frac{x}{\sqrt{1+x^2}} \)
- \( \frac{x}{\sqrt{1-x^2}} \)
- \( \sqrt{1-x^2} \)
- \( \frac{x}{x} \)
- \( \sqrt{x^2-1} \)
- \( x \)

\[ \sin^{-1}(x) = \theta \]

\[ \tan \theta = \frac{x}{\sqrt{1-x^2}} \]
11. Let \( f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (x - 2)^2 & \text{if } x \geq 1 \end{cases} \). Find the \( \lim_{x \to 1} f(x) \) if it exists.

-2
2
0
\( \square \) the limit does not exist
\( \square \) 1

\[ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 2)^2 = 1^2 = 1 \]

\[ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - 2)^2 = (1 - 2)^2 = (-1)^2 = 1 \]

\[ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \]

12. Which of the following statements is false?

- i. Polynomial functions are continuous for all real numbers.
- ii. Rational functions are continuous for all real numbers.
- iii. The \( \sin(x) \) and \( \cos(x) \) functions are continuous for all real numbers.
- iv. The tangent function is continuous for all values in its domain.
- v. If \( f(x) \) and \( g(x) \) are both continuous at a point, \( c \), then \( f(x) + g(x) \) will be continuous as point \( c \).

\( \square \) statement ii is false
\( \square \) statement i is false
\( \square \) none of the statements is false
\( \square \) statement v is false
\( \square \) statement iii is false
\( \square \) statement iv is false
Part II: Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

13

(a) Find the domain of: \( y = \sqrt{e^{2x} - 3} \).

\[ e^{2x} - 3 \geq 0 \]

\[ e^{2x} \geq 3 \]

\[ 2x \geq \ln 3 \]

\[ x \geq \frac{\ln 3}{2} \]

\( D: \left[ \frac{\ln 3}{2}, \infty \right) \)

(b) Find an equation for the inverse of this function: \( y = \frac{x + 1}{2x + 1} \).

\( x = \frac{y + 1}{2y + 1} \) 

\( x (2y + 1) = y + 1 \)

\( 2xy + x = y + 1 \)

\( 2xy - y = 1 - x \)

\( y (2x - 1) = 1 - x \)

\[ y = \frac{1 - x}{2x - 1} \]

(c) Find the limit: \( \lim_{x \to 1} \frac{\sqrt{x - 1}}{x - 1} \)

\[ \lim_{x \to 1} \frac{\sqrt{x - 1}}{x - 1} = \frac{\sqrt{x - 1}}{(\sqrt{x + 1})(\sqrt{x - 1})} = \frac{1}{\sqrt{x + 1}} \]

Also can solve by multiplying by conjugate.
On parts a–g, find the exact value of each expression. On part h, solve for $x$.

(a) $\tan \left( \frac{5\pi}{6} \right) = \frac{\sin \left( \frac{5\pi}{6} \right)}{\cos \left( \frac{5\pi}{6} \right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

(b) $\sin \left( \frac{-\pi}{3} \right) = \sin \left( \frac{5\pi}{3} \right) = \frac{-\sqrt{3}}{2}$

(c) $\sin^{-1} \left( \frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}$

(d) $\sec \left( \frac{\pi}{3} \right) = \frac{1}{\cos \left( \frac{\pi}{3} \right)} = \frac{1}{\frac{1}{2}} = 2$

(e) $\log_{16} 60 - \log_{16} 3 - \log_{16} 5 = \log_{16} \left( \frac{60}{15} \right) = \log_{16} 4 = \frac{1}{2}$

(f) $\ln \left( \frac{1}{e^2} \right) = \ln \left( \frac{1}{e} \right) = \ln \left( e^{-2} \right) = -2 - 2 \ln(e) = -2 - 2 = -4$

(g) $\log_{10}(1000) = 3$

(h) Solve for $x$: $\log_{10}(3x + 10) = 2$

\[ 10^2 = 3x + 10 \]
\[ 100 = 3x + 10 \]
\[ 2x = 90 \]
\[ x = 45 \]
Evaluate each of the following limits or indicate that the limit does not exist.

(a) \( \lim_{{x \to 0}} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right) = \frac{1}{x} - \frac{1}{x(x+1)} = \frac{(x+1)-1}{x(x+1)} = \frac{x}{x(x+1)} = \frac{1}{x+1} \)

\[ \text{Common denominator...} \]
\[ \lim_{{x \to 0}} \frac{1}{x+1} = \frac{1}{0+1} = 1 \]

(b) \( \lim_{{x \to 1}} \left( \frac{x^3 - 1}{x^2 - 1} \right) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{3x}{2} & \text{if } x > 0 \end{cases} \]
\[ \lim_{{x \to 1}} \frac{3x}{2} = \frac{3}{2} = \lim_{{x \to 1}} \frac{3x}{2} = \frac{3}{2} \]

(c) \( \lim_{{x \to -3}} \frac{2x + 6}{|x + 3|} = \frac{2(x+3)}{|x+3|} \)
\[ \lim_{{x \to -3}^+} \frac{2(x+3)}{(x+3)} = -2 \]
\[ \lim_{{x \to -3}^-} \frac{2(x+3)}{-(x+3)} = 2 \]

(d) \( \lim_{{x \to \infty}} \frac{\sqrt{x^2 + 4} - 3}{x} = \frac{\sqrt{x^2 + 4} - 3}{x} \)
\[ \lim_{{x \to \infty}^-} \frac{\sqrt{x^2 + 4} - 3}{x} = \lim_{{x \to \infty}^-} \frac{\sqrt{1+\frac{4}{x^2}} - 3}{1} = \frac{1-3}{1} = -2 \]
Given: \( \lim_{x \to 1} \frac{2 + 4x}{3} = 2 \). Find the largest possible number \( \delta \) so that when \( |x - 1| < \delta \), then \( \left| \frac{2 + 4x}{3} - 2 \right| < 1 \).

\[
\left| \frac{2 + 4x - 10}{3} \right| < 1 \quad \left| \frac{4x - 4}{3} \right| < 1 \quad \left| \frac{4}{3} \right| |x - 1| < 1 \quad |x - 1| < \frac{3}{4}
\]

\( \delta = \frac{3}{4} \)

\[
3 = \frac{2 + 4x}{3} \quad |1 - \frac{2 + 4x}{3}| < 1
\]

\[
9 = 2 + 4x \quad 3 = 2 + 4x
\]

\[
7 = 4x \quad 1 = 4x
\]

\[
\frac{7}{4} = x \quad \frac{1}{4} = x
\]

\[
\left| \frac{7}{4} - 1 \right| = \frac{3}{4} \quad \left| \frac{1}{4} - 1 \right| = \frac{3}{4}
\]

\( \delta = \frac{3}{4} \)

If \( |x - 1| < \frac{3}{4} \), then \( \left| \frac{2 + 4x}{3} - 2 \right| < 1 \)
Given the following piecewise function: \( f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ 0 & \text{if } 1 \leq x \leq 4 \\ x - 2 & \text{if } x > 4 \end{cases} \)

(a) Identify any values of \( x \) for which the function is discontinuous.

\[ \lim_{x \to 1^-} f(x) = 0 \]

(b) For any \( x \) values identified in part a, give a justification for why the function is discontinuous at that value of \( x \).

\[ \lim_{x \to 4^-} f(x) = 0 \\ \lim_{x \to 4^+} f(x) = (4 - 2) = 2 \]

\[ \lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x) \]

So, \( x = 4 \), \( f(x) \) not continuous
Show without graphing that the function \( f(x) = x^3 + 2x - 4 \) has a zero. Justify your answer.

\[ f(0) = -4 \]
\[ f(2) = 8 \]

Because \( f(x) \) is continuous,

By the Intermediate Value Theorem,

there exists at least one zero between

\( 0 < x < 2 \).
Consider the function: \( f(x) = 3x^2 - 4x + 1 \).

(a) Find \( f'(2) \) using the limit definition of the derivative. Full credit will not be given if the limit definition is not used.

\[
\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = \frac{3(2+h)^2 - 4(2+h) + 1 - 5}{h}
\]

\[
= \frac{12 + 12h + 3h^2 - 8 - 4h - 4}{h}
\]

\[
= \frac{3h^2 + 8h}{h}
\]

\[
= \frac{8 + 3h}{h}
\]

(b) Find the equation of the tangent line to the curve at the point where \( x = 2 \).

\[
f(x) = 3x^2 - 4x + 1
\]

\[
f'(x) = 6x - 4
\]

\[
f'(2) = 12 - 4 = 8
\]

\[
y = 8x + b
\]

\[
5 = 8(2) + b
\]

\[
5 = 16 + b
\]

\[
b = -11
\]

\[
y = 8x - 11
\]
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