Math 113
Exam 2
Oct 24, 25, 2016 (Late Day: Oct 26, 2016)

Encode your BYU ID in the grid below.

Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.

II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 5 points each.

III) For questions which require a written answer, show all your work in the space provided and justify your answer.

IV) Simplify your answers.

V) No books, notes, or calculators of any type are allowed.

VI) There is no time limit on this exam.
Part I: Multiple Choice Questions: Mark the correct answer. (5 points each)

1. Which of the following would be a partial fraction decomposition for the rational function

\[ \frac{x^2 + 3x + 1}{(x^2 + 1)(x^2 - 1)(x + 2)(x^2 + 4)} \]

- \[ \frac{A}{x^2 + 1} + \frac{B}{x^2 - 1} + \frac{C}{x + 2} + \frac{D}{x^2 + 4} \]
- \[ \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1} + \frac{D}{x - 1} + \frac{E}{x + 2} + \frac{F}{x^2 + 4} + \frac{G}{(x + 2)^2} + \frac{H}{x^2 + 4} \]
- \[ \frac{Ax + B}{x^2 + 1} + \frac{C}{x^2 - 1} + \frac{D}{x + 2} + \frac{Gx + H}{x^2 + 4} \]
- \[ \frac{x^2 + 3x + 1}{(x^2 + 1)(x^2 - 1)(x + 2)^2(x^2 + 4)} \]
- \[ \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2} + \frac{E}{x - 1} + \frac{F}{(x - 1)^2} + \frac{G}{x + 2} + \frac{H}{(x + 2)^2} + \frac{I}{x^2 + 4} + \frac{J}{x^2 + 4} \]
- \[ \frac{A}{x^2 + 1} + \frac{B}{x + 1} + \frac{C}{x - 1} + \frac{D}{x + 2} + \frac{E}{(x + 2)^2} + \frac{F}{x^2 + 4} \]

2. Suppose that \( p(t) \) is a probability density function for how long it takes you to get to school on a given morning (measured in minutes). The probability that it takes you less than ten minutes is .4, and the probability that it takes you less than five minutes is .2 (i.e. \( P(0 \leq T \leq 10) = .4 \) and \( P(0 \leq T \leq 5) = .2 \)). What is the probability that it takes you between 5 and 10 minutes.

- .2
- .08
- .6
- .1
- 1
- 0
- .2
3. Find the centroid of the region bounded by \( y = \frac{1}{x^2 + 1}, \ y = \frac{1}{x + 1}, \ x = 0, \ x = 1. \)

- \( \bar{x} = 2 - 2 \ln 2, \ \bar{y} = 0 \)
- \( \bar{x} = \frac{1}{2}, \ \bar{y} = \frac{1}{\ln 2} \)
- \( \bar{x} = \frac{1 - \ln 2}{\ln 2}, \ \bar{y} = 0 \)
- \( \bar{x} = \frac{1 - \ln 2}{\ln 2}, \ \bar{y} = \frac{1 - \ln 2}{2} \)
- \( \bar{x} = \frac{1}{2}, \ \bar{y} = 0 \)
- \( \bar{x} = 0, \ \bar{y} = \frac{1 - \ln 2}{\ln 2} \)
- \( \bar{x} = 1 - \ln 2, \ \bar{y} = \frac{1}{\ln 2} \)

\[
A = \int_0^1 \left( \frac{1}{x^2 + 1} - \frac{1}{x + 1} \right) \, dx
\]

\[
\bar{x} = \frac{1}{A} \int_0^1 x f(x) \, dx
\]

\[
\bar{y} = \frac{1}{2A} \int_0^1 y f(x) \, dx
\]

4. Find \( \int x \sec^2 x \, dx \)

- \( \ln |\sec x| + C \)
- This function does not have an antiderivative.
- \( x \sec x - \ln |\cos x| + C \)
- \( -\ln |\cos x| + C \)
- \( x \tan x + \ln |\cos x| + C \)
- \( x \sec^2 x - \tan x + C \)
- \( x \tan x + C \)
- \( x \tan x - \tan x + C \)

\[
\int x \sec^2 x \, dx = \frac{1}{4} \ln^2 \frac{1}{x^2} + \tan x + C
\]

\[
x \tan x = \int \tan x \, dx
\]

\[
x \tan x + \ln |\cos x| + C
\]
5 Suppose \( f(x) \) is a function with \( |f''(x)| < 6 \) on \([1, 5] \). If we use Midpoint to approximate \( \int_1^5 f(x) \, dx \), what is the smallest number of subintervals we can use to ensure that the approximation is within 0.1 of the actual integral? (Recall that the error bound for Midpoint is given by \( |E_M| \leq \frac{K(b-a)^3}{24n^2} \) where \( K \) is a bound on the absolute value of the second derivative.)

\[
|E_M| \leq \frac{K(5-1)^3}{24n^2}
\]

\[
|E_M| \leq \frac{6 \cdot 4^3}{24n^2} = \frac{64}{n^2} = 16
\]

6 If the curve \( y = x^2 \), \( 0 \leq x \leq \sqrt{2} \) is rotated about the y-axis, what is the resulting surface area?

\[
\text{Surface area: } 2\pi \int_0^{\sqrt{2}} x \sqrt{1 + 4x^2} \, dx
\]
7  Which ONE of the following improper integrals converges?

- \( \int_{0}^{\infty} \frac{1}{(5-x)^2} \, dx \)
- \( \int_{1}^{\infty} \frac{x+1}{x^2-x} \, dx \)
- \( \int_{0}^{\infty} \frac{x}{1+x^2} \, dx \)
- \( \int_{1}^{\infty} \frac{x^2+2x}{\sqrt{3x^7-x+1}} \, dx \)

8  Which of the following integrals represents the arc length of the curve \( y = \tan x \) from \( x = 0 \) to \( x = \frac{\pi}{4} \)?

- \( \int_{0}^{\pi/4} \sqrt{1 + \sec^4(x)} \cdot \sec^2 x \, dx \)
- \( \int_{0}^{\pi/4} \sqrt{1 + \sec^2(x)} \, dx \)
- \( \int_{0}^{\pi/4} \sqrt{1 + \sec^4(x)} \, dx \)
- \( \int_{0}^{\pi/4} 2\pi \tan x \sqrt{1 + \sec^2(x)} \, dx \)
- \( \int_{0}^{\pi/4} 2\pi \sqrt{1 + \sec^4(x)} \, dx \)
- \( \int_{0}^{\pi/4} 2\pi \tan x \sqrt{1 + \sec^4(x)} \, dx \)
- \( \int_{0}^{\pi/4} 2\pi \sqrt{1 + \sec^2(x)} \, dx \)
- \( \int_{0}^{\pi/4} \sqrt{1 + \sec^2(x)} \cdot \sec^2 x \, dx \)

\( \sqrt{1 + \left( \tan x \right)^2} \)
Part II: Justify your answer and show all work for full credit.

Let \( p \) be defined as following:

\[
p(x) = \begin{cases} 
  c \sin \frac{\pi x}{5}, & \text{if } 0 \leq x \leq 5 \\
  0, & \text{if } x < 0 \text{ or } x > 5
\end{cases}
\]

(a) For what value of \( c \) is \( p \) a probability density function?

\[
\int_0^5 c \sin \left( \frac{\pi x}{5} \right) \, dx = 1
\]

(b) For that value of \( c \) calculate the mean.

\[
\text{mean} = \mu = \int_{-5}^{5} x f(x) \, dx
\]
Calculate the following integral or show it diverges.

\[ \int_{0}^{\pi/2} \cos \theta \frac{d\theta}{\sqrt{\sin \theta}} = \lim_{b \to 0^+} \int_{b}^{\pi/2} \frac{du}{\sqrt{u}} = \lim_{b \to 0^+} \left[ -2u^{1/2} \right]_{b}^{\pi/2} = \left[ 2 \cdot \frac{1}{2} - 2 \cdot 0 \right] = 1 \]
Your bicycle has a speedometer. While riding one day, you clock your speed every 30 seconds (or half a minute) for 3 minutes, as recorded in the following table.

<table>
<thead>
<tr>
<th>time (minutes)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
<th>3/2</th>
<th>2</th>
<th>5/2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed (meters/minute)</td>
<td>150</td>
<td>200</td>
<td>210</td>
<td>230</td>
<td>200</td>
<td>130</td>
<td>100</td>
</tr>
</tbody>
</table>

a) Use the trapezoid rule to approximate the distance you traveled during the three minutes as accurately as possible. What is \( n \)?

\[
n = \frac{\pi}{6}
\]

\[
T_n = \frac{\pi}{2} \left( \frac{150 + 2 \cdot 200 + 2 \cdot 210 + 2 \cdot 230 + 2 \cdot 200 + 2 \cdot 130 + 100}{900} \right)
\]

\[
= \frac{\pi}{2} \left( \frac{150 + 400 + 420 + 460 + 400 + 260 + 100}{900} \right)
\]

\[
= \frac{\pi}{2} \left( \frac{570}{900} \right) = \frac{\pi}{2} \left( \frac{19}{30} \right) = \frac{5}{4} \left( \frac{109}{30} \right) = \frac{5}{4} \left( 3 \frac{19}{30} \right) = \frac{5}{4} \left( 3 \frac{19}{30} \right)
\]

b) Use Simpson's rule to approximate the distance you traveled.

\[
S_n = \frac{2310}{6}
\]
Find

\[ \int \frac{5x^2 - 2x + 15}{(x^2 + 5)(x - 1)} \, dx \]

**Partial Fraction Decomposition**

\[ \frac{Ax + B}{x^2 + 5} + \frac{C}{x - 1} \]

**plug in A, B, C**

\[ \int \frac{2x + 0}{x^2 + 5} \, dx + \frac{3}{x - 1} \, dx \]

\[ \int \frac{2x}{x^2 + 5} + \frac{3}{x - 1} \, dx \]

\[ u = x^2 + 5, \quad w = x - 1 \]

\[ du = 2x \, dx, \quad dw = dx \]

\[ \int \frac{du}{u} + \frac{3 \, dw}{w} \]

\[ \ln |x^2 + 5| + 3 \ln |x - 1| + c \]

\[ (Ax + B)(x - 1) + C(x^2 + 5) = 5x^2 - 2x + 15 \]

**distribute**

\[ Ax^2 - Ax + Bx - B + Cx^2 + 5C = 5x^2 - 2x + 15 \]

**make system of equations for similar variables.**

\[ A x^2 + C x^2 = 5 x^2 \]

\[ -A x + B x = -2 x \]

\[ -B + 5C = 15 \]

some like normal system of equations.

\[ A + C = 5 \]

\[ B = -2 + A \]

\[ -2 + A = 5C - 15 \]

\[ 5C - 15 = B \]

\[ A = 5C - 13 \]

\[ (5C - 13) + C = 5 \]

\[ 6C = 18 \]

\[ C = 3 \]

\[ A + 3 = 5 \]

\[ A = 2 \]

\[ B = -2 + 2 \]

\[ B = 0 \]
Find the length of the curve \( y = \frac{x^2}{4} - \frac{1}{2} \ln x \) from \( x = 1 \) to \( x = 2 \).

**Arc length:** 
\[
\int_{1}^{2} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

\[
y' = \frac{x}{2} - \frac{1}{2x}
\]

\[
(y')^2 = \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}
\]

\[
\int_{1}^{2} \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}} \, dx = \int_{1}^{2} \sqrt{\frac{1}{4}(x + \frac{1}{x})^2} \, dx
\]

Simplify:

\[
= \int_{1}^{2} \left( \frac{1}{2}x + \frac{1}{2} \ln x \right) \, dx
\]

\[
= \left[ \frac{1}{4}(x + \frac{1}{2} \ln^2 x) \right]_1^2
\]

\[
= \left[ \frac{3}{4} + \frac{1}{2} (\ln 2)^2 \right] - \left[ \frac{1}{4} + \frac{1}{2} \ln^2 1 \right]
\]

\[
= \frac{3}{4} + \frac{1}{2} (\ln 2)^2 - \frac{1}{4}
\]
A trough has ends that are in the shape of a right isosceles triangle (with one of the legs horizontal and the other vertical). If the legs of the isosceles triangle are 1 m long, and the trough is full of fluid, find the force due to pressure on one of the triangular ends. (Leave the weight density of the liquid as \( \rho g \).)

\[
\begin{align*}
\frac{1}{1} &= \frac{x}{1-y} \\
A &= \frac{1}{2} \\
&= -y \\
g &= \int (y) \cdot y \, dy \\
g &= \int y - y^2 \, dy \\
g &= \left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 \\
g &= \frac{1}{2} - \frac{1}{3} \\
g &= \frac{1}{6} \\
g &= \frac{1}{6} \\
&= \frac{1}{6}
\end{align*}
\]
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