NS 40107780 **001**

Math 113
Exam 2

Nov 6-7, 2017 (Late Day: Nov 8, 2017)

Name:____________________

Section:____________________

Instructor:____________________

Encode your BYU ID in the grid below.

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Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.

II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are worth up to 5 points each.

III) Multiple choice questions that have more than one correct answer will be marked with a ♣. All other multiple choice questions have only one correct answer.

IV) For questions which require a written answer, show all your work on the page where the question is, and justify your answer.

V) Simplify your answers.

VI) No books, notes, or calculators of any type are allowed.

VII) There is no time limit on this exam.

Part I: Multiple Choice Questions: *Maximum 5 points each. Questions marked with a ♣ have more than one correct answer. Mark* ***all*** *correct answers. The other questions have one correct answer.*

1 Consider the integral $I = \int_1^2 \frac{3}{x} dx$. If you approximate I with the Trapezoid Rule, what is the smallest number of intervals n necessary to guarantee that the error E_T is less than .01? Recall that the error bound for the Trapezoid Rule is given by $|E_T| \leq \frac{K(b-a)^3}{12n^2}$, and K is a constant satisfying $K \geq |f''(x)|$.

- ☐ 7
- ☐ 6
- ☐ 9
- ☐ 8
- ☐ 51
- ☐ 50

2 ♣ Which of the following improper integrals converges? (This question has more than one correct answer. Mark all correct answers.)

- ☐ $\displaystyle\int_0^\infty \frac{x}{1+x^2}\, dx$
- ☐ $\displaystyle\int_0^\infty e^{-x}\, dx$
- ☐ $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}+x^2}\, dx$
- ☐ $\displaystyle\int_1^\infty \frac{\sin^2(x)+1}{\sqrt{x}}\, dx$
- ☐ $\displaystyle\int_1^\infty \frac{x^2+2x}{\sqrt{3x^7-x+1}}\, dx$

3 ♣ Which of the following integrals represents the **surface area** obtained by revolving the curve defined by $y = \arctan x$ from $x = 0$ to $x = 1$ about the y-axis? (This question has more than one correct answer. Mark all correct answers.)

- ☐ $\int_0^{\pi/4} 2\pi y \sqrt{1 + \sec^4(y)}\, dy$
- ☐ $\int_0^1 2\pi x \sqrt{1 + (\frac{1}{1+x^2})}\, dx$
- ☐ $\int_0^1 2\pi \arctan x \sqrt{1 + (\frac{1}{1+x^2})^2}\, dx$
- ☐ $\int_0^1 2\pi x \sqrt{1 + (\frac{1}{1+x^2})^2}\, dx$
- ☐ $\int_0^{\pi/4} 2\pi \tan(y) \sqrt{1 + \sec^4(y)}\, dy$

4 Which integral gives the arclength of $y = x^2$ from $x = 0$ to $x = 1$.

- ☐ $\int_0^1 \sqrt{1 + x^4}\, dx$
- ☐ $\int_0^1 \sqrt{1 + \sqrt{y}}\, dy$
- ☐ $\int_0^1 \sqrt{1 + 4x^2}\, dx$
- ☐ $\int_0^1 \sqrt{1 + 2x^2}\, dx$
- ☐ $\int_0^1 2\pi x \sqrt{1 + 4x^2}\, dx$
- ☐ $\int_0^1 2\pi x^2\, dx$

5 Find the arc length of the curve from $t = 0$ to $t = 1$ given by the parametric equations below.

$$x = e^t \cos(t) \quad y = e^t \sin(t)$$

- ☐ $\sqrt{2}(e-1)$
- ☐ $e-1$
- ☐ $\sqrt{2}(e+1)$
- ☐ $e+1$
- ☐ $\sqrt{2}$

6 ♣ Which of the following give polar coordinates (r, θ) for the point whose Cartesian coordinates are $(-2, -2\sqrt{3})$. (This question has more than one correct answer. Mark all correct answers.)

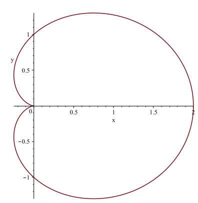
- ☐ $(4, \frac{\pi}{3})$
- ☐ $(4, \frac{4\pi}{3})$
- ☐ $(4, \frac{2\pi}{3})$
- ☐ $(-2-2\sqrt{3}, \frac{-\pi}{3})$
- ☐ $(-4, \frac{\pi}{3})$

7 Which ONE of the following points is NOT on the curve defined by the parametric equations

$$x = t^2 + 1, \quad y = \arctan t \qquad \text{for } -\infty < t < \infty.$$

- [ ] $(2, \frac{\pi}{4})$
- [ ] $(2, -\frac{\pi}{4})$
- [ ] $(4, \frac{\pi}{3})$
- [ ] $(\frac{4}{3}, \frac{\pi}{6})$
- [ ] $(2, \frac{\pi}{6})$

8 Find the area inside the cardioid defined in polar coordinates by the equation $r = 1 + \cos\theta$. A graph of the cardioid is given below.



- [ ] $\frac{3\pi}{2}$
- [ ] 3π
- [ ] $\frac{3\pi}{4}$
- [ ] π
- [ ] 2π

Part II: *Justify your answer and show all work for full credit.*

9 ☐**0** ☐**1** ☐**2** ☐**3** ☐**4** ☐**5** ☐**6** *Administrative Use Only*

You have a wooden log 30m long. In order to approximate the volume of the log, you measure the radius of the log at 5m intervals, and they are recorded in the table below.

Length from one end (m)	0	5	10	15	20	25	30
Radius (m)	.05	.06	.07	.08	.09	.1	.15

a. (*2 points*) Use the Midpoint rule with $n = 3$ to write down the terms of a sum that approximates *volume* of the log. You do not need to evaluate the sum. It may be helpful to write an integral that represents the volume of the log.

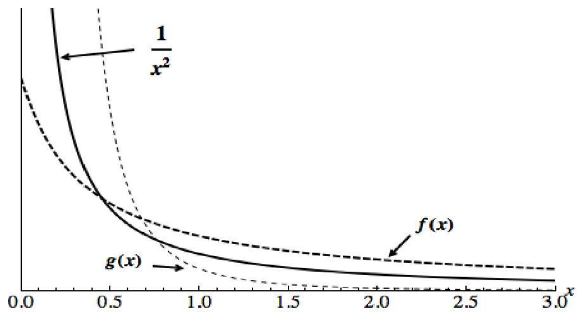
M_3=____

b. (*4 points*) Use the Simpson's rule with $n = 6$ to write down the terms of a sum that approximates *volume* of the log. You do not need to evaluate the sum.

S_n=____

10 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 *Administrative Use Only*

Consider the functions f and g (pictured below) satisfying the following statements:



$$\frac{1}{x^2} \leq g(x) \text{ for } 0 < x < \tfrac{1}{2}$$
$$\frac{1}{x^2} \geq g(x) \text{ for } x > 1$$
$$\frac{1}{x^2} \leq f(x) \text{ for } x > 1$$

a. (*3 points*) Using the information about $f(x)$ and $g(x)$ given above, write DIVERGES if the integral diverges, and CONVERGES if the integral converges. If there is not enough information to decide, write NOT ENOUGH INFORMATION.

$$\int_1^\infty f(x)\,dx$$
$$\int_1^\infty g(x)\,dx$$
$$\int_0^1 g(x)\,dx$$

b. (*6 points*) Calculate the following integral or show it diverges.

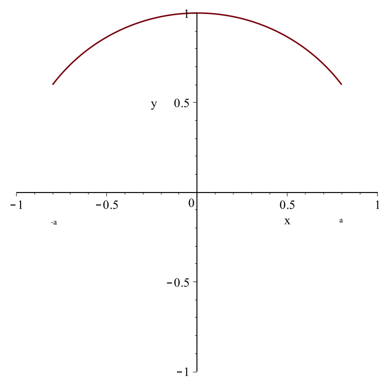
$$I = \int_0^{\pi/2} \tan(x)dx$$

11 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** $\square$ **8** $\square$ **9** *Administrative Use Only*

Find a point (x_0, y_0) on the curve $y = \frac{2}{3}(x+2)^{3/2}$ so that the arc length from $(-2, 0)$ to (x_0, y_0) is $2\sqrt{3} - \dfrac{2}{3}$.

 0 1 2 3 4 5 6 7 *Administrative Use Only*

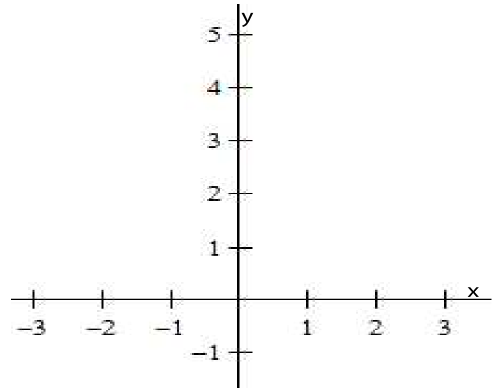
Let a be some number satisfying $0 < a \leq 1$. The arc (pictured below) of the unit circle $x^2 + y^2 = 1$ above the x–axis and between $-a \leq x \leq a$ is rotated about the x-axis. Compute the resulting surface area.



13 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 *Administrative Use Only*

A trough is filled to the top with water. The ends of the trough are shaped like the region bounded by the curves $y = x^2$ and $y = 4$.

a. (*1 point*) Draw a picture of the region (one end of the trough) on the axes below.



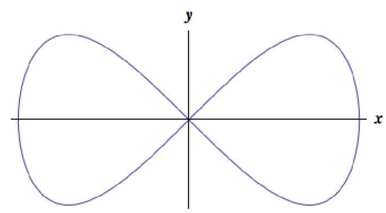
b. (*4 points*) Set up an integral that expresses the hydrostatic force from the water on one end of the trough. Use ρ for the density of water, and g for the acceleration due to gravity. Leave your integral in terms of ρ and g.

c. (*4 points*) Evaluate the integral from part **b**. Leave your answer in terms of ρ and g.

□ **0** □ **1** □ **2** □ **3** □ **4** □ **5** *Administrative Use Only*

This problem concerns the curve represented by the parametric equations given below. There are four parts to this problem. Two are on this page and two are on the next page.

$$x = 2\cos t \quad y = \sin(2t) \qquad \text{for } 0 \le t \le 2\pi$$



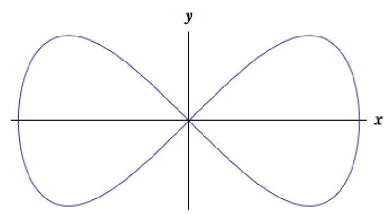
a. (*2 points*) At which values of t does the curve pass through the origin?

b. (*3 points*) Find all points on the curve where the curve has a horizontal tangent line.

15 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** *Administrative Use Only*

This problem is a continuation of the problem on the previous page that concerns the curve represented by the parametric equations

$$x = 2\cos t \quad y = \sin(2t) \qquad \text{for } 0 \le t \le 2\pi.$$



c. (*3 points*) Find the equation of the tangent line at the point $(1, -\frac{\sqrt{3}}{2})$.

d. (*3 points*) Write an integral that expresses the length of this curve. You do not need to evaluate this integral.

16 $\square$ **0** $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** $\square$ **8** $\square$ **9** *Administrative Use Only*

Find the area of the shaded region in the first quadrant that lies inside the curve $r = 2 + \sin(2\theta)$ and outside the circle $r = \frac{5}{2}$, as shown in the graph below.