Selected Formulas

Cross Products: \( a \times (b \times c) = (a \cdot c)b - (a \cdot b)c \)

Quadratic Surfaces in \( \mathbb{R}^3 \)

Cone: \( x^2/a^2 + y^2/b^2 = z^2/c^2 \)

Ellipsoid: \( x^2/a^2 + y^2/b^2 + z^2/c^2 = 1 \)

Elliptic Paraboloid: \( x^2/a^2 + y^2/b^2 = z/c \)

Hyperbolic Paraboloid: \( x^2/a^2 - y^2/b^2 = z/c \)

Hyperboloid of One Sheet: \( x^2/a^2 + y^2/b^2 - z^2/c^2 = 1 \)

Hyperboloid of Two Sheets: \( x^2/a^2 - y^2/b^2 - z^2/c^2 = 1 \)

Local Behavior of Curves

\( T(t) = \frac{r'(t)}{|r'(t)|} \)

\( \kappa = \frac{|T'(t)|}{|r'(t)|} \)

\( \kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} \)

\( \kappa = \frac{|f''(x)|}{|1 + (f'(x))^2|^{3/2}} \)

\( N(t) = \frac{T'(t)}{|T'(t)|} \)

Motion in Space

\( a = v'T + \kappa v^2 N \)

\( v' = (r'(t) \cdot r''(t))/|r'(t)| \)

\( \kappa v^2 = |r'(t) \times r''(t)|/|r'(t)| \)
True/False and Multiple Choice

1. Which of the following are consistent with the right hand rule?
   A. The positive x axis points straight up. The positive y axis points north. The positive z axis points east.
   B. The positive x axis points straight up. The positive y axis points north. The positive z axis points west.
   C. The positive x axis points straight up. The positive y axis points east. The positive z axis points north.
   D. The positive x axis points east. The positive y axis points south. The positive z axis points straight down.
   E. The positive x axis points south. The positive y axis points straight up. The positive z axis points east.
   F. The positive x axis points south. The positive y axis points straight down. The positive z axis points east.

2. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|$
   A. True  B. False

3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
   A. True  B. False

4. If $\theta$ is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ then $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$
   A. True  B. False

5. If $\theta$ is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ then $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$
   A. True  B. False

6. Vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel whenever $\mathbf{a} \times \mathbf{b} = 0$  A. True  B. False

7. Which are true?
   A. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$
   B. $\mathbf{i} \times \mathbf{j} = -\mathbf{k}$
   C. $\mathbf{j} \times \mathbf{k} = \mathbf{i}$
   D. $\mathbf{j} \times \mathbf{k} = -\mathbf{i}$
   E. $\mathbf{k} \times \mathbf{i} = \mathbf{j}$
   F. $\mathbf{k} \times \mathbf{i} = -\mathbf{j}$
   G. $\mathbf{j} \times \mathbf{i} = \mathbf{k}$
   H. $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$
   I. $\mathbf{k} \times \mathbf{j} = \mathbf{i}$
   J. $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$
   K. $\mathbf{i} \times \mathbf{k} = \mathbf{j}$
   L. $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$

8. $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$
   A. True  B. False

9. $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$
   A. True  B. False
Free Response

1. Find the length of the parametric space curve \langle t^2, 4t, 4ln(-t) \rangle where \(-4 \leq t \leq -1\)

2. Simplify the following: \( a \cdot [(a + b) \times c] + [c \times (a + b)] \cdot a \)

3. Find three different surfaces which contain the space curve \( r(t) = \langle 2t, e^t, e^{2t} \rangle \)

4. Find the velocity and position vectors of a particle with the given acceleration, and the given initial velocity and position.

   (a) \( a(t) = \langle 2, 0, 2t \rangle, v(0) = \langle 1, 0, 0 \rangle, r(0) = \langle 0, 1, 0 \rangle \)

   (b) \( a(t) = \sin(t)i + 2\cos(t)j + 6tk, v(0) = -k, r(0) = j - 4k \)
5. Where is the curvature maximized for $y = e^x$?

6. Find equations for the normal plane and osculating plane of the parametric curve $\mathbf{r}(t) = (\ln(t), 2t, t^3)$ at the point $(0, 2, 1)$.

7. Find the curvature of the following:
   (a) $\mathbf{r}(t) = t^3 \mathbf{j} + t^2 \mathbf{k}$

   (b) $\mathbf{r}(t) = \sqrt{6}t^2 \mathbf{i} + 2t \mathbf{j} + 2t^3 \mathbf{k}$

   (c) $\mathbf{r}(t) = (t, t^2, t^3)$ at the point $(1,1,1)$.

   (d) $y = x^4$

   (e) $y = xe^x$

   (f) $y = xe^x$ at the point $(1,e)$. 
8. Find the velocity, acceleration and speed functions of a particle with the given position functions, and evaluate them at the given time $t$:

(a) $\mathbf{r}(t) = (-\frac{1}{2}t^2, t), \quad t = 2$

(b) $\mathbf{r}(t) = 3\cos(t)i + 2\sin(t)j, \quad t = \pi/3$

(c) $\mathbf{r}(t) = ti + t^2j + 2k, \quad t = 1$

(d) $\mathbf{r}(t) = \sqrt{2}i + e^tj + e^{-t}k, \quad t = 2$

(e) $\mathbf{r}(t) = e^t(\cos(t)i + \sin(t)j + tk), \quad t = \pi$
9. Give an equation of the plane perpendicular to the vector \( \langle 2, 1, -1 \rangle \) which passes through the point \( (1, 2, 4) \)

10. Give a parametric representation of the line parallel to the \( x \)-axis and passes through the point \( (3, 4, 5) \)

11. Evaluate the following limits, if they exist. If they do not exist, explain why not.

(a) 
\[
\lim_{(x,y)\to(3,-1)} \frac{x^2y - xy^3 + 5}{x^2 - 3y^2 + xy - 6}
\]

(b) 
\[
\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}
\]

(c) 
\[
\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}
\]

(d) 
\[
\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}
\]

(e) 
\[
\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2} + 1 - 1}
\]

(f) 
\[
\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4 + y^4}
\]

(g) 
\[
\lim_{(x,y,z)\to(\pi,0,1/3)} e^{yz} \tan(xz)
\]