Math 314 Test 1, Example Problems #1

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June 25, 2020

These questions are not from previous exams. These are questions that cover many, but not necessarily all, of the concepts which could appear on the exams. If you feel there is an error with the solutions, please contact the Math Lab via mathlabupper@mathematics.byu.edu, and we will rectify the mistake.

Selected Formulas

Cross Products: $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$

Quadratic Surfaces in $\mathbb{R}^3$

Cone: $x^2/a^2 + y^2/b^2 = z^2/c^2$
Ellipsoid: $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$
Elliptic Paraboloid: $x^2/a^2 + y^2/b^2 = z/c$
Hyperbolic Paraboloid: $x^2/a^2 - y^2/b^2 = z/c$
Hyperboloid of One Sheet: $x^2/a^2 + y^2/b^2 - z^2/c^2 = 1$
Hyperboloid of Two Sheets: $x^2/a^2 - y^2/b^2 - z^2/c^2 = 1$

Local Behavior of Curves

$T(t) = \frac{r'(t)}{|r'(t)|}$

$\kappa = \frac{|T'(t)|}{|r'(t)|}$

$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$\kappa = \frac{|f''(x)|}{[1+(f'(x))^2]^{\frac{3}{2}}}$

$N(t) = \frac{T'(t)}{|T'(t)|}$

Motion in Space

$\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}$

$v' = (\mathbf{r'}(t) \cdot \mathbf{r''}(t))/|\mathbf{r'}(t)|$

$\kappa v^2 = |\mathbf{r'}(t) \times \mathbf{r''}(t)|/|\mathbf{r'}(t)|$

True/False and Multiple Choice

1. Which of the following are consistent with the right hand rule?
   - A. The positive x axis points straight up. The positive y axis points north. The positive z axis points east.
   - B. The positive x axis points straight up. The positive y axis points north. The positive z axis points west.
   - C. The positive x axis points straight up. The positive y axis points east. The positive z axis points north.
   - D. The positive x axis points east. The positive y axis points south. The positive z axis points straight down.
   - E. The positive x axis points south. The positive y axis points straight up. The positive z axis points east.
   - F. The positive x axis points south. The positive y axis points straight down. The positive z axis points east.

2. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|$

   A. True B. False

3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

   A. True B. False

4. If θ is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ then $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$

   A. True B. False

5. If θ is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ then $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$

   A. True B. False

6. Vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel whenever $\mathbf{a} \times \mathbf{b} = 0$ A. True B. False

7. Which are true?
   - A. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$
   - B. $\mathbf{i} \times \mathbf{j} = -\mathbf{k}$
   - C. $\mathbf{j} \times \mathbf{k} = \mathbf{i}$
   - D. $\mathbf{j} \times \mathbf{k} = -\mathbf{i}$
   - E. $\mathbf{k} \times \mathbf{i} = \mathbf{j}$
   - F. $\mathbf{k} \times \mathbf{i} = -\mathbf{j}$
   - G. $\mathbf{j} \times \mathbf{i} = \mathbf{k}$
   - H. $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$
   - I. $\mathbf{k} \times \mathbf{j} = \mathbf{i}$
   - J. $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$
   - K. $\mathbf{i} \times \mathbf{k} = \mathbf{j}$
   - L. $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$

8. $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$

   A. True B. False

9. $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$

   A. True B. False

Free Response

1. Find the length of the parametric space curve $\langle t^2, 4t, 4ln(-t)\rangle$ where $-4 \leq t \leq -1$

2. Simplify the following: $a \cdot [(a+b) \times c] + [c \times (a+b)] \cdot a$

3. Find three different surfaces which contain the space curve $r(t) = \langle 2t, e^t, e^{2t}\rangle$

4. Find the velocity and position vectors of a particle with the given acceleration, and the given initial velocity and position.
   (a) $\mathbf{a}(t) = \langle 2, 0, 2t\rangle$, $\mathbf{v}(0) = \langle 1, 0, 0\rangle$, $\mathbf{r}(0) = \langle 0, 1, 0\rangle$

   (b) $\mathbf{a}(t) = \sin(t)\mathbf{i} + 2\cos(t)\mathbf{j} + 6t\mathbf{k}$, $\mathbf{v}(0) = -\mathbf{k}$, $\mathbf{r}(0) = \mathbf{j} - 4\mathbf{k}$

5. Where is the curvature maximized for $y = e^x$?

6. Find equations for the normal plane and osculating plane of the parametric curve $\mathbf{r}(t) = \langle ln(t), 2t, t^2 \rangle$ at the point $(0, 2, 1)$.

7. Find the curvature of the following:

   (a) $\mathbf{r}(t) = t^3\mathbf{j} + t^2\mathbf{k}$

   (b) $\mathbf{r}(t) = \sqrt{6}t^2\mathbf{i} + 2t\mathbf{j} + 2t^3\mathbf{k}$

   (c) $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point (1,1,1).

   (d) $y = x^4$

   (e) $y = xe^x$

   (f) $y = xe^x$ at the point (1,e).

8. Find the velocity, acceleration and speed functions of a particle with the given position functions, and evaluate them at the given time t:

   (a) $\mathbf{r}(t) = \langle -\frac{1}{2}t^2, t \rangle, \quad t = 2$

   (b) $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}, \quad t = \pi/3$

   (c) $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 2\mathbf{k}, \quad t = 1$

   (d) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}, \quad t = 2$

   (e) $\mathbf{r}(t) = e^t(\cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}), \quad t = \pi$

9. Give an equation of the plane perpendicular to the vector $\langle 2, 1, -1\rangle$ which passes through the point $\langle 1, 2, 4\rangle$

10. Give a parametric representation of the line parallel to the x-axis and passes through the point $\langle 3, 4, 5\rangle$

11. Evaluate the following limits, if they exist. If they do not exist, explain why not.

    (a)
    $$\lim_{(x,y)\to(3,-1)} \frac{x^2y - xy^3 + 5}{x^2 - 3y^2 + xy - 6}$$

    (b)
    $$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

    (c)
    $$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

    (d)
    $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$

    (e)
    $$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$$

    (f)
    $$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4 + y^4}$$

    (g)
    $$\lim_{(x,y,z)\to(\pi,0,1/3)} e^{y^2} \tan(xz)$$