Do not discuss this exam with any person. Do not use notes. Do not use a calculator. Do not use a book.

Remember that \( \log \) with no base written means \( \log_{10} \) and that \( \ln \) is \( \log_e \).

**Problems**

**Question 1.** Simplify the expression. \( \frac{12x^2(y^{-2}z^{\frac{1}{2}})^3}{2^2x^3y^{-4}\sqrt{z}} \)

\[
\begin{array}{ccc}
  a) & \frac{3z}{xy^3} & b) \frac{3}{xy^2} & c) \frac{3z^2}{y^2} \\
  d) & \frac{z}{xy^2} & e) \frac{3z}{xy^2} & f) \frac{3z^2}{xy^2}
\end{array}
\]

**Question 2.** Let \( f(x) = \ln(x) \) and \( g(x) = x^2 - 1 \). Find the domain of the composite function \( (f \circ g)(x) \).

\[
\begin{array}{ccc}
  a) & (-\infty, 0) \cup (0, \infty) & b) & (-\infty, 0) \cup (1, \infty) & c) & (-\infty, -1) \cup (0, \infty) \\
  d) & (-\infty, -1) \cup (1, \infty) & e) & (-\infty, -2) \cup (2, \infty) & f) & (-\infty, -1) \cup (2, \infty)
\end{array}
\]

**Question 3.** Let \( f(x) = \frac{1}{x} \) and \( g(x) = x^2 - 1 \). Find the domain of the composite function \( (f \circ g)(x) \).

\[
\begin{array}{ccc}
  a) & \text{All real numbers} & b) & \text{All non-negative real numbers} & c) & (-\infty, -1) \cup (-1, 1) \cup (1, \infty) \\
  d) & (-\infty, -1) \cup (-1, 0) \cup (0, \infty) & e) & (-\infty, -1) \cup (-1, \infty) & f) & (-\infty, 1) \cup (1, \infty)
\end{array}
\]

**Question 4.** Let \( f(x) = 10(2^x) \). Find \( f^{-1}(x) \).

\[
\begin{array}{ccc}
  a) & 10 \log_2(x) & b) & \log_{10}(\frac{x}{2}) & c) & \log_2(\frac{x}{10}) \\
  d) & \log_2(\frac{10}{x}) & e) & \log_{10}(\frac{x}{10}) & f) & \log_2(x)
\end{array}
\]

**Question 5.** Let \( f(x) = x + 3, \ g(x) = x^2 + 3 \). Find \( (f \circ g)(x) + (g \circ f)(x) \).

\[
\begin{array}{ccc}
  a) & x^2 + 6 & b) & x^2 + 9 & c) \frac{x + 3}{x^2 + 3} \\
  d) & 2x^2 + 6x + 18 & e) & 2x^2 + 6x + 15 & f) & 2x^2 + 6x + 12
\end{array}
\]

**Question 6.** Let \( f(x) = \frac{x + 1}{2x - 2} \). Find \( f^{-1} \).

\[
\begin{array}{ccc}
  a) & y = \frac{x + 1}{x - 1} & b) & y = \frac{3x + 1}{3x - 1} & c) & y = \frac{x + 1}{-2x + 1} \\
  d) & y = \frac{2x + 1}{2x - 1} & e) & y = \frac{2x + 1}{2x - 2} & f) & y = \frac{2x - 2}{x + 1}
\end{array}
\]
Question 7. Solve the equation. $\ln(x - 3e^2) = 3$

$$a) \ x = e^3 + 3e^2 \\
\quad b) \ x = e^3 + 2e^2 \\
\quad c) \ x = e^3 + 3e \\
\quad d) \ x = e \\
\quad e) \ 3 + e^2 \\
\quad f) \ x = 2e^3 + e^2$$

Question 8. Write as a single logarithm with coefficient 1. $\ln(x + 3) + \ln(x - 4)$

$$a) \ \ln(12x^3) \\
\quad b) \ \ln(x^2 + x + 12) \\
\quad c) \ \ln(x^2 - x - 12) \\
\quad d) \ 3\ln(12x) \\
\quad e) \ -\ln(-x^2 + x + 12) \\
\quad f) \ -\ln(x^2 - x - 12)$$

Question 9. Solve the equation. $\ln(x + 1) - \ln(x + 4) = \ln(x + 3) - \ln(x + 8)$

$$a) \ x = 5 \\
\quad b) \ x = 4 \\
\quad c) \ x = 1 \\
\quad d) \ x = 2 \\
\quad e) \ x = 3 \\
\quad f) \ x = 0$$

Question 10. Solve the equation. $e^{-2x} = 9$

$$a) \ \frac{\ln(9)}{3} \\
\quad b) \ -\frac{\ln(9)}{2} \\
\quad c) \ -\frac{\ln(2)}{9} \\
\quad d) \ -\frac{\ln(3)}{9} \\
\quad e) \ \frac{\ln(3)}{2} \\
\quad f) \ -\frac{\ln(4)}{9}$$

Question 11. Simplify the expression. $\log_7(9) \log_3(1/4) \log_2(\sqrt{7})$

$$a) \ -\sqrt{2} \\
\quad b) \ -2 \\
\quad c) \ -\frac{1}{2} \\
\quad d) \ \sqrt{2} \\
\quad e) \ 2 \\
\quad f) \ \frac{1}{2}$$

Question 12. Simplify the expression. $\log(160) + \log(25) - \log(4)$

$$a) \ -2 \\
\quad b) \ 3 \\
\quad c) \ 4 \\
\quad d) \ 2 \\
\quad e) \ -3 \\
\quad f) \ -4$$

Question 13. Write as a single logarithm. $\log_5(7) + 2\log_5(3) - \frac{1}{2}\log_5(49) + 3\log_5(2)$

$$a) \ \log_5(56) \\
\quad b) \ \log_5(72) \\
\quad c) \ \log_5(36) \\
\quad d) \ \log_5(64) \\
\quad e) \ \log_5(48) \\
\quad f) \ \log_5(144)$$

Question 14. Find the sum of the solutions of the equation. $16x - 5(4^x) + 6 = 0$

$$a) \ \frac{1}{9} + \log_4(5) \\
\quad b) \ \frac{1}{2} + \log_4(3) \\
\quad c) \ \frac{1}{3} + \log_4(9) \\
\quad d) \ \frac{1}{9} + \log_4(2) \\
\quad e) \ \frac{1}{4} + \log_4(12) \\
\quad f) \ \frac{1}{6} + \log_4(6)$$
Question 15. Solve for $y$. $\log_2(y) = 3x^3 + \frac{1}{2} \log_2(C)$

\[
\begin{align*}
a) y &= C2^{3x^3} \\
b) y &= C^22^{7x} \\
c) y &= \sqrt{C}2^{7x} \\
d) y &= C^22^{3x} \\
e) y &= \sqrt{C}2^{9x} \\
f) y &= \sqrt{C}2^{3x^3}
\end{align*}
\]

Question 16. How many years does it take for an amount of money to triple at 2% interest compounded continuously?

\[
\begin{align*}
a) 50 \ln(3) \\
b) 25 \ln(3) \\
c) \frac{\ln(3)}{50} \\
d) -50 \ln(3) \\
e) -25 \ln(3) \\
f) \frac{\ln(3)}{25}
\end{align*}
\]

Question 17. The price of a car decays at a rate of 10% per year. How long does it take for the car to lose 50% of its value?

The equation for the depreciation of a car is $A = Pe^{-rt}$. $P$ is the current value of the car, $P$ is the original price of the car, $r$ is the rate of depreciation and $t$ is the number of years.

\[
\begin{align*}
a) 10 \ln(2) \\
b) \ln(2) \\
c) \frac{\ln(2)}{10} \\
d) -\ln(2) \\
e) -\ln(2) \\
f) -10 \ln(2)
\end{align*}
\]

Question 18. Which of the following parabolas is the same as $y = \frac{1}{4}x^2 + \frac{1}{2}x - \frac{15}{4}$?

\[
\begin{align*}
a) 8(y - 2) &= (x + 3)^2 \\
b) 4(y - 4) &= (x - 1)^2 \\
c) (y - 4) &= (x - 1)^2 \\
d) 4(y + 2) &= (x - 3)^2 \\
e) 4(y + 4) &= (x + 1)^2 \\
f) (y + 2) &= (x - 3)^2
\end{align*}
\]

Question 19. What is the $x$-coordinate of the vertex of the parabola $4y - 28 + 8x = 4x^2 + 4$?

\[
\begin{align*}
a) -1 & \\
b) 1 & \\
c) -2 & \\
d) 2 & \\
e) 0 & \\
f) 3
\end{align*}
\]

Question 20. What is the $y$-coordinate of the focus of the parabola $4(y - 7) = (x - 2)^2$?

\[
\begin{align*}
a) 5 & \\
b) 6 & \\
c) 7 & \\
d) 8 & \\
e) 9 & \\
f) 10
\end{align*}
\]
### Key

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