Math 541 F17: Homework Problems

HW 1. Due Friday September 15 at 4 p.m.

I.1 (pp49-50 Ed.1; pp48-49 Ed.2): 1.1, 1.2, 1.3, 1.7, 1.10

I.4 (pp51-52 Ed.1; p50 Ed.2): 4.1, 4.2 (the first part only where you are asked to show a collection of sets is a topology)

I.9 (p56 Ed.1; pp57 Ed.2): 9.2(v)

I:10 (p58 Ed.1; p58 Ed. 2): 10.1 (i)(ii)(iii) (assume in (i) that not both α and β are zero; ignore the piece in (ii) about $\overline{A}$ or $\overline{B}$ being compact)

I:13 From Lecture Note #2, Homework problems

2A. Prove that the collection $\mathcal{B}$ of open balls $B_\rho(x)$ in a metric space satisfies the two conditions of Proposition 4.1.

2B. In a metric space $\{X; d\}$ prove that each singleton set $\{x\}$ is closed.

HW 2. Due Friday September 22 at 4 p.m.

I:13 (p59 Ed.1; p59 Ed.2): 13.3, 13.5

I:16 From Lecture Note 3, the homework problems

3A. Give a proof of Corollary 16.2 (on p.46 of Ed.1; this is Corollary 16.1 in Ed.2).

3B. Give a proof of Proposition 16.2c (on p.63 in Ed.1, on p.65 in Ed.2).

II:2 (p110 Ed.1; pp108-109 Ed.2): 2.2, 2.3 (Ignore last part of 2.2 in Ed.2 that starts with "Set $D_1 = E_1$ and $D_{n+1} = D_n \triangle E_{n+1}$ ... ")

II:3 (p111 Ed.1): 3.3 or (p109 Ed.2) 3.2 (with $\mu(E) = \infty$ replaced by $\mu(E) = 1$ when E has countable complement, as this makes it more interesting).

II:3 From Lecture Note #5, the homework problems

5A. For a measure μ on a σ-algebra $\mathcal{A}$, prove that if $\{E_n\}$ in $\mathcal{A}$ is monotone increasing and $E = \cup E_n$, then $\mu(E_n) \to \mu(E)$ as $n \to \infty$.

5B. For a measure μ on a σ-algebra $\mathcal{A}$, prove that if $\{E_n\}$ in $\mathcal{A}$ is monotone decreasing, there exists $k \in \mathbb{N}$ such that $\mu(E_k) < \infty$, and $E = \cap E_n$, then $\mu(E_n) \to \mu(E)$ as $n \to \infty$; show that this is false if there is no $k \in \mathbb{N}$ such that $\mu(E_k) < \infty$.

HW 3. Due Friday September 29 at 4 p.m.

II.3 From Lecture Note #6, the homework problems

6A. Give an example of a measure μ for which $\mu(B - A) = \mu(B) - \mu(A)$ fails when $\mu(A) = \infty$.

6B. If $\{\mu_\alpha : \alpha \in I\}$ is a finite or countable collection of measures on the same σ-algebra $\mathcal{A}$, then $\sum \mu_\alpha$ is a measure on $\mathcal{A}$.

II.4: (p111 in Ed.1, p112 in Ed.2), 4.1, 4.3

II:4 and II.5 From Lecture Note #7, the homework problems

7A. For $f(x) = e^x$, find $\mu_{f,e}\big((a, b)\big)$.

7B. For E a square of unit edge in $\mathbb{R}^2$, prove that $\mathcal{H}_{\alpha,\epsilon}(E) = 0$ for all $\alpha > 2$.