SUPPLEMENTAL HOMEWORK PROBLEMS

7A: Let $R$ be an integral domain with quotient field $F$. If $T$ is an integral domain with $R \subset T \subset F$, prove that $F$ is isomorphic to the quotient field of $T$.

7B: Let $R$ be an integral domain, and for each maximal ideal $M$, consider $R_M$ as a subring of the field of fractions of $R$. Prove that
\[ \bigcap_{M \text{ maximal}} R_M = R. \]

7C: Let $R = \mathbb{Z}_6$ and $S = \{\bar{2}, \bar{4}\} \subset R$. Prove that $S^{-1}R$ is a field with three elements.

7D: Prove that a commutative ring with $1 \neq 0$ is local if and only if for all $r, s \in R$, $r + s = 1$ implies that $r$ or $s$ is a unit.