Homework 21

1. Give a one line proof that no finite field is algebraically closed.

2. Prove:

   **Theorem.** Let $K/F$ be a normal algebraic extension and let $f \in F[x]$ be an irreducible polynomial. If $\alpha, \beta \in K$ are roots of $f$, then there is a field isomorphism $\sigma : K \to K$ with $\sigma|_F = 1_F$ and $\sigma(\alpha) = \beta$.

3. Prove:

   **Theorem.** Let $K/F$ be an algebraic extension. The following are equivalent.
   (a) $K/F$ is normal.
   (b) If $\bar{F}$ is an algebraic closure of $F$ containing $K$, then any injective ring homomorphism $\sigma : K \to \bar{F}$ that is the identity on $F$ has image equal to $K$. 
