Homework 38

1. Let $L/K$ be an algebraic Galois extension, let $G = \text{Gal}(L/K)$ and let $H$ be a closed subgroup of $G$. Prove that $L_H/K$ is Galois if and only if $H \triangleleft G$.

2. Recall the following definition from topology:

   **Definition.** A topological space $X$ is Hausdorff if, given any two points $x, y \in X$ there are open sets $U_x$ and $U_y$, with $x \in U_x$, $y \in U_y$, and $U_x \cap U_y = \emptyset$.

   Let $L/K$ be an algebraic Galois extension. Prove that $G = \text{Gal}(L/K)$ with the Krull topology is a Hausdorff topological space.

3. Prove that a closed subset of a compact topological space is compact.