(1) Let $R$ be a commutative ring with 1, and let $I_1, \ldots, I_n$ be pairwise relatively prime ideals in $R$ (i.e. $I_i + I_j = R$ for all $i \neq j$). Let

$$J = \bigcap_{i=1}^{n} I_i.$$ 

Prove that

(a) $J = \prod_{i=1}^{n} I_i$. (Hint, do this for $n = 2$ first, and then use induction.)

(b) $R/J \cong R/I_1 \times \cdots \times R/I_n$. (Hint: Use the first isomorphism theorem.)

(2) Use Problem (1) to prove: Let $R$ be a commutative ring with 1, let $I_1, \ldots, I_n$ be pairwise relatively prime ideals in $R$, and let $x_1, \ldots, x_n \in R$. Then there is an element $x \in R$ such that $x \equiv x_i \pmod{I_i}$ for all $1 \leq i \leq n$.

(3) Prove that if $R$ is a Dedekind domain with only finitely many prime ideals, then $R$ is a principal ideal domain.