First Test Math 485
Oct. 8th to 10th, 2018

1. Solve the following problems using a computing device (sage server, lap-top, slide-rule, abacus or soroban):
   (a) Find gcd(50523933123707, 134778956009).
   (b) Factor the integer 136876323412974601 into prime powers.
   (c) Compute $23911^{33051}$ modulo 771223.
   (d) Solve the equation $233x = 98797$ in the field of integers modulo 112997 (a prime).

2. Decrypt the following affinely coded message:
   ZUBYNZYQOBGBZDOGVIBTVSTBUBUBLQYUVUbYQHDBQGVOTYGBNEZTVYVIBVUVYBLBOT
   (Hint: The first three letters represent the English word for the first positive integer.)

3. Solve the following problems in the field of order 49 (using the tables handed out):
   (a) Solve the equation $x^3 = 1$. (Hint: Factor out $x - 1$ and solve the quadratic.)
   (b) Compute $7^3$ in the field.
   (c) Find all solutions to the equation $x^3 = 21$, using parts (a) and (b).

4. The bit sequence 00110101111000100... satisfies a linear (mod 2) recurrence of no more than length 6. Find the recurrence, and find the next three binary digits of the sequence.

5. Suppose $s \equiv 2x + 4y \pmod{23}$ and $t \equiv 3x + 5y \pmod{23}$. Find integers $a, b, c, d$ so that $x \equiv as + bt \pmod{23}$ and $y \equiv cs + dt \pmod{23}$.

6. Find all integers $x$ such that $x \equiv 11 \pmod{25}$ and $2x \equiv 7 \pmod{27}$ simultaneously.

*Explain your methods in each problem. Answers which appear out of thin air may be discredited.*