In this project you will compare the function $f(x) = \sin x$ to its Taylor polynomials (in $x$) $P_1(x), P_3(x), P_5(x), P_7(x),$ and $P_9(x)$.

**Exercise 1** Plot $f(x)$ and $P_n(x)$ ($n = 1, 3, 5, 7, 9$) on the entire interval $[0, 5]$. When you look at this particular plot, what is the largest interval of the form $[0, b]$ on which the graph of $P_n(x)$ is indistinguishable from the plot of $f(x)$? (Your answer should be five different intervals, one for each of these five values of $n$.)

**Exercise 2** Use Taylor’s Theorem and the Lagrange form of the remainder to find intervals of the form $[0, b]$ (with $b$ as large as you can) on which $|P_n(x) - f(x)|$ is guaranteed to be less than or equal to 0.01. (Your answer should be five different intervals, one for each of the five values of $n$.)

**Exercise 3** Using a computer or calculator, approximate the largest interval of the form $[0, b]$ on which $|P_n(x) - f(x)|$ actually is less than 0.01. (Your answer should be five different intervals, one for each of the five values of $n$.) By what factor did your application of Taylor’s Theorem in Exercise 2 underestimate the size of these intervals in each case?

Higher order Taylor polynomials in $x$ do not always give better approximations of $f$ than lower order Taylor polynomials in $x$ when $x$ is far from 0.

**Exercise 4** For $n = 1, 3, 5,$ and 7, approximate the smallest positive value of $x$ for which $|P_{n+2}(x) - f(x)| \geq |P_n(x) - f(x)|$.

No Maple worksheet is provided for this project. If you choose to use Maple to do this project, here are some hints you might find helpful:

- **Maple** commands end with semicolons and are executed by hitting the enter key.
- To evaluate $\sin x$, type “sin(x);” and hit the enter key. For example, to evaluate $\sin(0.86)$, type “sin(0.86);” and hit the enter key to see that the answer is .7578425629 (to 10 decimal places).
- The expression $3x^2 + 6x + 7$ should be typed “3*x^2+6*x+7”.
- To define a function $g$ having the formula $g(x) = x^4 - 7x$, you can type
  
  “g := x -> x^4-7*x;”

  and then hit the enter key. You can then apply this function to just about anything. For example, typing “g(0.45);” and hitting the enter key will then produce the output $-3.10899375$. 

• “abs(x)” means |x|.

• To plot the function f on the interval [a, b], type “plot(f(x), x=a..b);” and hit the enter key. For example, typing “plot(abs(x), x=-1..2);” and hitting the enter key will plot |x| for x ∈ [-1, 2].

• You can plot several functions on the same set of axes. For example, typing “plot([abs(x), sin(x), g(x)], x=-1..3);” and hitting the enter key will plot |x|, sin x, and g(x) for x ∈ [-1, 3].

• More information on various Maple commands can be found on the Help menu within Maple.