1. (b) global minimum at $(-1, 2)$. (f) global maximum at $\left(\frac{1}{\sqrt{2}}, \frac{1}{4}\right)$. (j) global maximum at $(3, \frac{8}{3})$. (n) global minimum at the origin.

2. (b) global maximum at $\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$, global minimum at $(-1, -1)$, and local minimum at $(1, 1)$.
   
   (c) global maximum of $\sqrt{2}$ at $\frac{\pi}{4} + 2n\pi$, global minimum of $-\sqrt{2}$ at $\frac{5\pi}{4} + 2n\pi, n \in \mathbb{Z}$.
   
   (f) local maximum at $(-\frac{\pi}{3} + 2n\pi, -\frac{\pi}{3} + 2n\pi + \sqrt{3})$, local minimum at $(\frac{\pi}{3} + 2n\pi, \frac{\pi}{3} + 2n\pi - \sqrt{3})$.
   
   (j) global minimum at $(\frac{1}{3}, -\frac{2}{3e})$.
   
   (n) no extrema

3. The function $f(x) = \begin{cases} x - a, & a \leq x < \frac{1}{2}(a + b) \\ -\frac{1}{2}(x - b), & \frac{1}{2}(a + b) \leq x \leq b \end{cases}$ is defined on $[a, b]$ but has no maximum there.

4. $f''$ changes sign at $c$, so $(f')'$ changes sign at $c$ and hence $f'$ has an extremum.

5. Theorem. If $f''(c) = 0$ and $f'''(c) \neq 0$, then $f$ has an inflection point at $c$.
   
   Proof. If $f''(c) = 0$, then $c$ is a critical point of $f'$ at which the derivative is zero, so we may apply the second derivative test to $f'$. If $(f')''(c) \neq 0$, then $f'$ has an extremum at $c$, so that the derivative of $f'$ changes at $c$ and hence the concavity changes at $c$. Thus $c$ is an inflection point of $f$.

6. Since $f$ is continuous on a closed interval, it has a global maximum there, which is also a local maximum, so occurs at $c$.

7. Suppose $f$ is defined in $(a, b)$ and $c \in (a, b)$ is the only critical point of $f$, and $f$ has a local maximum at $c$ (the case of minimum is similar). Since $f$ is continuous and $c$ is the only critical point, $f$ is differentiable in $(a, b)$ except possibly at $c$. If $f$ were to have a value greater than $f(c)$ somewhere, then it must have the same value at some point, and then Rolle’s Theorem would give us another critical point, which we can’t have. Thus the value at $c$ is a global maximum.

8. (a) P2 should maximize $S$ with respect to $y$, getting $y = \frac{27 - x}{4}$.
   
   (b) P1 should replace $y$ in $S$ with the value that P2 is going to use, and minimize the resulting $S$, getting $x = -1$.
   
   (c) P1 should minimize $S$ with respect to $x$, getting $x = \frac{y - 9}{2}$.
   
   (d) P2 should replace $x$ in $S$ with the value that P1 is going to use and maximize the resulting $S$, getting $y = 7$.
   
   (e) Each player has to “go first” so P1 chooses $x = -1$ and P2 chooses $y = 7$. (The resulting score is 0, so the game is uninteresting, even though it is fair.)