1. (a) \{1, 3, 5, 7, 2\}. (b) \(LHS(4) = \frac{11}{4}, RHS(4) = \frac{13}{4}\). (c) \(LHS(8) = \frac{23}{8}, RHS(8) = \frac{25}{8}\). (d) \{1, 1 + \frac{1}{n}, 1 + \frac{2}{n}, \ldots, 1 + \frac{n-1}{n}, 2\}. \(LHS(n) = 2 + \frac{2}{n^2}[1 + 2 + 3 + \cdots + (n-1)], RHS(n) = 2 + \frac{n}{n^2}[1 + 2 + \cdots + n]\). (e) \(LHS(n) = 3 - \frac{1}{n}, RHS(n) = 3 + \frac{1}{n}\). (f) 3 (g) The trapezoid has bases 2 and 4 and altitude 1, so has area 3.

2. (a) \(\pi\). (b) \(LHS(n)\) will be an overestimate, \(RHS(n)\), an underestimate, because function is decreasing. (c) 3.58422; 2.25089. (d) 3.39532; 2.72865. (e) Yes; \(LHS(n)\) should decrease as \(n\) increases, and \(RHS(n)\) should increase.

3. (a) Because the function is increasing, \(LHS(n)\) is an underestimate and \(RHS(n)\) is an overestimate. (b) \(LHS(3) = 0.715249, RHS(3) = 1.238848\). (c) \(LHS(6) = 0.863382, RHS(6) = 1.125182\). (d) As \(n\) increases, underestimates should increase and overestimates should decrease. That is what we see here. (e) about 1.

4. \(A \approx \frac{\pi}{2}\).

5. (a) There is no inscribed polygon, so the inner area is zero. (b) The smallest circumscribed polygon is a square with area 1, so the outer area is 1.

7. Proof: Suppose that \(N - S\) is non-empty. Then \(N - S\) has a least element \(m\). Now 1 \(\in S\), so \(m \neq 1\). Hence \(m - 1 \in S\), since \(m\) is the least element not in \(S\), so \(m = (m - 1) + 1 \in S\), a contradiction. Therefore \(S = \emptyset\).

8. (a) Let \(S = \{n \in N \mid 1 + 2 + \cdots + n = \frac{n(n+1)}{2}\} = 1 \in S\) since 1 = \(\frac{1(1+1)}{2}\).

Suppose \(k \in S\). Then \(1 + 2 + \cdots + k = \frac{k(k+1)}{2}\). Hence \(1 + 2 + \cdots + k + k + 1 = \frac{k(k+1)}{2} + (k + 1) = \frac{(k+1)(k+1)}{2} + (k + 1) = \frac{(k+1)(k+2)}{2} + (k + 1)\), and \(k + 1 \in S\). Hence \(S = \emptyset\), and the given statement is true for all positive integers \(n\).

(b) Let \(S = \{n \in N \mid 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\} = 1 \in S\) because \(1 = \frac{1(1+1)(2+1)}{6}\). Suppose \(k \in S\). Then \(1^2 + 2^2 + \cdots + k^2 + (k + 1)^2 = \frac{k(k+1)(2k+1)}{6} + (k + 1)^2 = \frac{(2k+1)[(k+1)^2 + g(k+1)]}{6} + (k + 1) = \frac{(k+1)(k+2)(2k+3)}{6} + (k + 1)\), and \(k + 1 \in S\). Hence \(S = \emptyset\), and the statement is true for all positive integers \(n\).

(e) Let \(S = \{n \in N \mid 8^n - 3^n\) is divisible by 5\}. Then \(8^1 - 3^1 = 5\) is divisible by 5, so \(1 \in S\). Suppose \(k \in S\). Then \(8^k - 3^k\) is divisible by 5. Hence \(8^{k+1} - 3^{k+1} = 8^k - 3 \cdot 8^k + 3 \cdot 8^k - 3^{k+1} = 8^k(8 - 3) + 3(8^k - 3^k)\) is divisible by 5 because each term is. Hence \(k + 1 \in S\), so \(S = \emptyset\) and \(8^n - 3^n\) is divisible by 5 for every positive integer \(n\).

10. If \(\delta = \frac{\epsilon}{4}\), then \(|x_1 - x_2| < \delta \Rightarrow |x_1^2 - x_2^2| = |x_1 - x_2||x_1 + x_2| < \frac{\epsilon}{4} \cdot 4 = \epsilon\), because \(x_1, x_2 < 2\).

11. (a) If \(f\) is uniformly continuous, then given \(\epsilon > 0\) there exists \(\delta > 0\) such that if \(p_1, p_2 \in [a, b]\) and \(|p_1 - p_2| < \delta\) then \(|f(p_1) - f(p_2)| < \frac{\epsilon}{b-a}\), by the definition in Problem 10.

(b) If \(\frac{b-a}{n} < \delta\), then let \(M_i\) and \(m_i\) be the points in the \(i\)th subinterval at which \(f\) has its maximum and minimum, respectively. Then \(f(M_i) - f(m_i) = f(M_i) - f(m_i) < \epsilon\).
\( f(m_i) < \frac{\epsilon}{b-a}, \) by part (a), so that

\[
US(n) - LS(n) = \frac{f(M_1) + \cdots + f(M_n)}{n} - \frac{f(m_1) + \cdots + f(m_n)}{n} \frac{b-a}{n}
\]

\[
< \frac{\epsilon}{b-a} + \cdots \frac{\epsilon}{b-a} \frac{b-a}{n}
\]

\[
= n \cdot \frac{\epsilon}{b-a} \cdot \frac{b-a}{n}
\]

\( = \epsilon. \)

(c) If \( US(n) - LS(n) < \epsilon, \) then as \( n \to \infty, \) \( US(n) - LS(n) \to 0. \) That means \( US(n) \) and \( LS(n) \) have the same limit, so \( R \) has area.