Name:_____

Student ID:_____

Instructor: Jason Grout

Math 343-1 (Linear Algebra with Applications)
Test 1
12–16 May 2006

Instructions:

- Notes, books, and calculators are not allowed.
- For multiple choice and true/false questions, put your answer in the blank provided.
- For questions which require a written answer, show all your work. In order to earn full credit, you will need to *neatly* show the work necessary to justify your answer on these pages.
- If an answer box is provided, put your answer in it.
- Simplify your answers.
- Should you have need for more space than is provided to answer a question, use the blank sides of adjacent pages and indicate this fact. Do not attach extra pages.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned
MC	6	
3	8	
4	8	
5	8	
6	8	
7	8	
8	4	
Sub	50	

#	Possible	Earned
9	4	
10	4	
11	18	
12	8	
13	8	
14	8	
Sub	50	
Total	100	

1 Multiple Choice

_____ 1. Which of the following is not an elementary matrix?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

_____ 2. For what values of α, if any, will $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & \alpha \\ 1 & \alpha & 1 \end{vmatrix}$ be 0?

(a) No value will work (b) 0 (c) 1

(d) 2 (e) 3 (f) 4

2 Computation

3. (8 points) The *coefficient* matrix A of a linear system has been reduced to

$$A = \begin{bmatrix} 1 & 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Solve $A\mathbf{x} = \mathbf{0}$. If there is no solution, say so. Otherwise, give all solutions.

4. (8 points) Compute the determinant

$$\begin{vmatrix} 1 & 0 & 0 & 2 \\ 1 & 1 & -2 & -1 \\ 0 & -7 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix}$$

5. (8 points) Row reduce the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & 0 & -1 \\ 1 & 2 & -3 & 3 \\ 3 & 1 & 1 & -1 \end{bmatrix}$ to reduced row echelon form.

6. (8 points) If

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix},$$

find A^{-1} or show that it does not exist.

7. (8 points) Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \\ -3 & 2 & 1 \end{bmatrix}$$

Find the LU factorization of A.

$L =$

$U =$

3 Short Answer

8. (4 points) What is the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 5 \end{bmatrix}$?

9. (4 points) If $\det(A) = 3$ and $\det(B) = 5$, then what is $\det(A^{-1}B^T)$?

10. (4 points) Which of the following matrices is invertible?

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & -1 \\ 5 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 1 & 2 \\ 1 & -2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & -4 \\ 0 & 3 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

4 True/False

11. (18 points) For each of the following statements, if the statement is always true, write "True" in the blank and write a short justification. If the statement is false, write "False" in the blank and give a counterexample. Let A and B be $n \times n$ matrices.

_____(a) If a system of linear equations has free variables, then it must be consistent.

_____(b) If a system of linear equations has more than one solution, it has infinitely many solutions.

_____(c) $(A + B)(A - B) = A^2 - B^2$.

_____(d) If $AB = A$, then $B = I$.

_____(e) If $AB = 0$, then either $A = 0$ or $B = 0$.

_____(f) If C is the matrix obtained from D by multiplying a row of D by 2, then $\det(C) = 2\det(D)$.

5 The rest of the test

12. (8 points) Prove the following: If B is an $n \times n$ skew symmetric matrix, where n is odd, then B is singular. (**Hint:** Recall that an $n \times n$ matrix B is skew symmetric if $B = -B^T$)

13. (8 points) Prove or disprove: If $\det(A - B) = 0$, then $A = B$.

14. (8 points) Let A and B be $n \times n$ matrices such that $AB = 0$. Prove that $(BA)^2 = 0$.