

Name: _____

Student ID: _____

Instructor: Jason Grout

Math 343-1 (Linear Algebra with Applications)

Test 2

30 May – 3 June 2006

Instructions:

- Notes, books, and calculators are not allowed.
- For multiple choice and true/false questions, put your answer in the blank provided.
- For questions which require a written answer, show all your work. In order to earn full credit, you will need to *neatly* show the work necessary to justify your answer on these pages.
- If an answer box is provided, put your answer in it.
- Simplify your answers.
- Should you have need for more space than is provided to answer a question, use the blank sides of adjacent pages and indicate this fact. Do not attach extra pages.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned	#	Possible	Earned
MC	15		11c	4	
6	3		11d	4	
7	3		12	21	
8	5		13	4	
9a	3		14	4	
9b	3		15	6	
10	5		16	6	
11a	4		17	6	
11b	4				
Sub	45		Sub	55	
			Total	100	

1 Multiple Choice

_____ 1. Which of the following is a true statement?

- (a) It is possible to have two different bases of a vector space with different numbers of basis elements.
- (b) $\text{Span} \left\{ \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \begin{pmatrix} -4 \\ 12 \end{pmatrix} \right\} = \mathbb{R}^2$
- (c) There are infinitely many subspaces of dimension 2 in \mathbb{R}^3 .
- (d) It is possible to find two subspaces of \mathbb{R}^n that do not intersect.
- (e) All lines in \mathbb{R}^3 are subspaces in \mathbb{R}^3 .
- (f) A vector space is, by definition \mathbb{R}^n or one of its subsets.

_____ 2. Which of the following sets span \mathbb{R}^3 ?

- (a) $\left\{ \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \right\}$
- (b) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$
- (c) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$
- (d) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix} \right\}$
- (e) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$
- (f) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}, \begin{pmatrix} -4 \\ 8 \\ -8 \end{pmatrix} \right\}$

_____ 3. Which of the following is a linear transformation from $\mathbb{R}^{n \times n}$ into $\mathbb{R}^{n \times n}$?

- (a) $L(A) = -3A$
- (b) $L(A) = A^{-1}$.
- (c) $L(A) = (A - A^T)^{-1}$
- (d) $L(A) = A^2$
- (e) $L(A) = I + A$
- (f) $L(A) = A - A^{-1}$

_____ 4. Which set of vectors is not linearly independent in P_4 ?

- (a) $x^3 + 2x^2, 2x^2 + x, x + 1, x^3 + 4x^2 + 4$ (b) $1 + 2x + 3x^2 + 4x^3, 1, x, x^2$
(c) $1 - 2x + x^2, 1 + 2x + x^2, x - x^2, x + x^2$ (d) $x^2 + x + 1, x + 1, 1$
(e) x, x^2, x^3 (f) $1 + x, 1 - x, x^3$

_____ 5. If

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \mathbf{v}_4 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix},$$

then what is the dimension of the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$?

- (a) 0 (b) 1 (c) 2
(d) 3 (e) 4 (f) 5

2 Computation and Short Answer

6. (3 points) In the vector space P_3 , find the coordinate vector $[\mathbf{v}]_B$ of $\mathbf{v} = x^2 - 2x + 1$ relative to the basis $B = [x + 1, 1, x^2 + 1]$.

7. (3 points) If A is an 8×5 matrix, what is the smallest possible dimension of the null space of A ? Give a brief justification for your answer.



8. (5 points) Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear operator that rotates a vector counterclockwise 90° , followed by a reflection about the y -axis. Find the standard matrix representing L .



9. Let $D: P_4 \rightarrow P_4$ given by $D(p) = p''$, the second derivative of p .

(a) (3 points) What is the kernel of D ?

(b) (3 points) What is the image of D ?

10. (5 points) Let

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + x_3 \\ x_2 - x_3 \end{pmatrix},$$

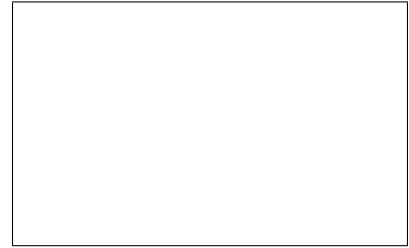
$$B = \left[\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right], \quad \text{and} \quad C = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right].$$

Find the matrix representing L from basis B to basis C .

11. The row reduced echelon form of

$$A = \begin{bmatrix} -1 & 3 & -5 & 4 & 18 \\ 1 & -2 & 4 & 0 & -7 \\ 2 & 0 & 4 & -3 & -8 \\ 5 & 1 & 9 & 2 & 2 \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) (4 points) Find a basis for the row space of A .



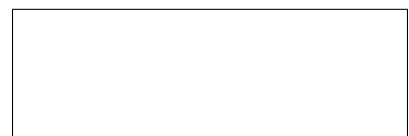
(b) (4 points) Find a basis for the column space of A .



(c) (4 points) Find a basis for the null space of A .



(d) (4 points) Find the rank of A .



3 True/False

12. (21 points) For each of the following statements, if the statement is always true, write “True” in the blank and write a short justification. If the statement is false, write “False” in the blank and give a counterexample.

_____ (a) Let V be a subspace of R^n . If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_j\}$ is a linearly independent set in V and $\{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_k\}$ is a spanning set for V , then $j < k$.

_____ (b) If A is an $m \times n$ matrix with $m < n$, then the dimension of the row space of A is less than the dimension of the column space of A .

_____ (c) If A is an $m \times n$ matrix, then the rank of A and the nullity of A sum to m .

_____ (d) Let A be an $n \times n$ matrix. The linear system $A\mathbf{x} = \mathbf{b}$ is consistent and has exactly one solution if and only if the nullity of A is 0.

_____ (e) If A is the standard matrix representing a linear transformation $L: R^n \rightarrow R^m$, then the null space of A is equal to the kernel of L .

_____ (f) If A and B are similar matrices, then $\text{rank}(A) = \text{rank}(B)$.

16. (6 points) Let A and B be invertible $n \times n$ matrices. Show that if A is similar to B , then A^{-1} is similar to B^{-1} .

17. (6 points) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that

$$\text{rank}(AB) \leq \text{rank}(A) \quad \text{and} \quad \text{rank}(AB) \leq \text{rank}(B).$$