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Student ID: ____________________

Instructor: Jason Grout

Math 343-1 (Linear Algebra with Applications) Test 2

30 May – 3 June 2006

Instructions:

- Notes, books, and calculators are not allowed.
- For multiple choice and true/false questions, put your answer in the blank provided.
- For questions which require a written answer, show all your work. In order to earn full credit, you will need to *neatly* show the work necessary to justify your answer on these pages.
- If an answer box is provided, put your answer in it.
- Simplify your answers.
- Should you have need for more space than is provided to answer a question, use the blank sides of adjacent pages and indicate this fact. Do not attach extra pages.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned
MC	15	
6	3	
7	3	
8	5	
9a	3	
9b	3	
10	5	
11a	4	
11b	4	
Sub	45	

#	Possible	Earned
11c	4	
11d	4	
12	21	
13	4	
14	4	
15	6	
16	6	
17	6	
Sub	55	
Total	100	

1 Multiple Choice

_____ 1. Which of the following is a true statement?

(a) It is possible to have two different bases of a vector space with different numbers of basis elements.

(b) $\text{Span}\left\{\begin{pmatrix}1\\-3\end{pmatrix}, \begin{pmatrix}-4\\12\end{pmatrix}\right\} = \mathbb{R}^2$

(c) There are infinitely many subspaces of dimension 2 in $\mathbb{R}^3$.

(d) It is possible to find two subspaces of $\mathbb{R}^n$ that do not intersect.

(e) All lines in $\mathbb{R}^3$ are subspaces in $\mathbb{R}^3$.

(f) A vector space is, by definition $\mathbb{R}^n$ or one of its subsets.

_____ 2. Which of the following sets span $\mathbb{R}^3$?

(a) $\left\{\begin{pmatrix}2\\1\\-3\end{pmatrix}, \begin{pmatrix}3\\-3\\-1\end{pmatrix}, \begin{pmatrix}1\\-4\\2\end{pmatrix}\right\}$

(b) $\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix}\right\}$

(c) $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\-3\end{pmatrix}, \begin{pmatrix}4\\2\\0\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix}\right\}$

(d) $\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\4\\6\end{pmatrix}, \begin{pmatrix}3\\6\\9\end{pmatrix}\right\}$

(e) $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}\right\}$

(f) $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}1\\-2\\2\end{pmatrix}, \begin{pmatrix}3\\3\\6\end{pmatrix}, \begin{pmatrix}-4\\8\\-8\end{pmatrix}\right\}$

_____ 3. Which of the following is a linear transformation from $\mathbb{R}^{n\times n}$ into $\mathbb{R}^{n\times n}$?

(a) $L(A) = -3A$

(b) $L(A) = A^{-1}$.

(c) $L(A) = (A - A^T)^{-1}$

(d) $L(A) = A^2$

(e) $L(A) = I + A$

(f) $L(A) = A - A^{-1}$

______ 4. Which set of vectors is not linearly independent in P_4?

(a) $x^3 + 2x^2, 2x^2 + x, x + 1, x^3 + 4x^2 + 4$
(b) $1 + 2x + 3x^2 + 4x^3, 1, x, x^2$
(c) $1 - 2x + x^2, 1 + 2x + x^2, x - x^2, x + x^2$
(d) $x^2 + x + 1, x + 1, 1$
(e) x, x^2, x^3
(f) $1 + x, 1 - x, x^3$

______ 5. If

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix},$$

then what is the dimension of the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5

2 Computation and Short Answer

6. (3 points) In the vector space P_3, find the coordinate vector $[\mathbf{v}]_B$ of $\mathbf{v} = x^2 - 2x + 1$ relative to the basis $B = [x + 1, 1, x^2 + 1]$.

7. (3 points) If A is an 8×5 matrix, what is the smallest possible dimension of the null space of A? Give a brief justification for your answer.

8. (5 points) Let $L\colon R^2 \to R^2$ be the linear operator that rotates a vector counterclockwise $90°$, followed by a reflection about the y-axis. Find the standard matrix representing L.

9. Let $D\colon P_4 \to P_4$ given by $D(p) = p''$, the second derivative of p.

(a) (3 points) What is the kernel of D?

(b) (3 points) What is the image of D?

10. (5 points) Let

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + x_3 \\ x_2 - x_3 \end{pmatrix},$$

$$B = \left[\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right], \quad \text{and} \quad C = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right].$$

Find the matrix representing L from basis B to basis C.

11. The row reduced echelon form of

$$A = \begin{bmatrix} -1 & 3 & -5 & 4 & 18 \\ 1 & -2 & 4 & 0 & -7 \\ 2 & 0 & 4 & -3 & -8 \\ 5 & 1 & 9 & 2 & 2 \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) (4 points) Find a basis for the row space of A.

(b) (4 points) Find a basis for the column space of A.

(c) (4 points) Find a basis for the null space of A.

(d) (4 points) Find the rank of A.

3 True/False

12. (21 points) For each of the following statements, if the statement is always true, write "True" in the blank and write a short justification. If the statement is false, write "False" in the blank and give a counterexample.

______(a) Let V be a subspace of R^n. If $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_j\}$ is a linearly independent set in V and $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k\}$ is a spanning set for V, then $j < k$.

______(b) If A is an $m \times n$ matrix with $m < n$, then the dimension of the row space of A is less than the dimension of the column space of A.

______(c) If A is an $m \times n$ matrix, then the rank of A and the nullity of A sum to m.

______(d) Let A be an $n \times n$ matrix. The linear system $A\mathbf{x} = \mathbf{b}$ is consistent and has exactly one solution if and only if the nullity of A is 0.

______(e) If A is the standard matrix representing a linear transformation $L \colon R^n \to R^m$, then the null space of A is equal to the kernel of L.

______(f) If A and B are similar matrices, then $\text{rank}(A) = \text{rank}(B)$.

4 The rest of the test

13. (4 points) Prove or disprove: The set of all singular 2×2 matrices is a subspace of $R^{2\times 2}$.

14. (4 points) Let A be an $n \times n$ matrix and let S be the set of all vectors $\mathbf{v}$ in R^n such that $A\mathbf{v} = \mathbf{v}$. Prove or disprove: S is a subspace of R^n.

15. (6 points) Use the definition of linear independence to prove that any subset of a linearly independent set of vectors is linearly independent.

16. (6 points) Let A and B be invertible $n \times n$ matrices. Show that if A is similar to B, then A^{-1} is similar to B^{-1}.

17. (6 points) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that

$$\text{rank}(AB) \leq \text{rank}(A) \qquad \text{and} \qquad \text{rank}(AB) \leq \text{rank}(B).$$