Name:________________________________

Student ID:______________________________

Instructor: Jason Grout

Math 343-1 (Linear Algebra with Applications) Test 3

13–15 June 2006

Instructions:

- Notes, books, and calculators are not allowed.
- For multiple choice and true/false questions, put your answer in the blank provided.
- For questions which require a written answer, show all your work. In order to earn full credit, you will need to *neatly* show the work necessary to justify your answer on these pages.
- If an answer box is provided, put your answer in it.
- Simplify your answers.
- Should you have need for more space than is provided to answer a question, use the blank sides of adjacent pages and indicate this fact. Do not attach extra pages.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned
MC	9	
4	5	
5	5	
6a	5	
6b	5	
6c	5	
7a	3	
7b	3	
7c	5	
8a	4	
Sub	49	

#	Possible	Earned
8b	4	
9	3	
10	4	
11	4	
12	12	
13	6	
14	6	
15	6	
16	6	
Sub	51	
Total	100	

1 Multiple Choice

______ 1. What is the angle between the vectors $(1, -1, -2, 1)^T$ and $(1, 1, 1, -2)^T$?

(a) $\cos^{-1}(-\frac{4}{7})$ (b) $\cos^{-1}(\frac{4}{7})$ (c) $\cos^{-1}(-\frac{4}{\sqrt{7}})$

(d) $\cos^{-1}(\frac{4}{\sqrt{7}})$ (e) $\cos^{-1}(-4)$ (f) $\cos^{-1}(4)$

(g) none of the above

______ 2. Which statement is not true?

(a) The null space of A is the orthogonal complement of the column space of A^T

(b) $S \cap S^{\perp} = \{\mathbf{0}\}$

(c) $(S^{\perp})^{\perp} = S$

(d) For any proper subspace $S \in \mathbb{R}^n$, there is an $\mathbf{x} \in \mathbb{R}^n$ that is not in either S or its orthogonal complement.

(e) For any proper subspace $S \in \mathbb{R}^n$, there is an $\mathbf{x} \in \mathbb{R}^n$ that cannot be written as a sum of vectors in S and its orthogonal complement.

(f) The dimension of S and the dimension of its orthogonal complement sum to the dimension of the space.

(g) none of the above

______ 3. Let V be a subspace of $\mathbb{R}^3$. Which of the following is a norm on $\mathbb{R}^3$?

(a) $||\mathbf{x}|| = x_1 + x_2 + x_3$ (b) $||\mathbf{x}|| = \max\{x_1^3, x_2^3, x_3^3\}$ (c) $||\mathbf{x}|| = \max\{|x_1|, |x_2|, |x_3|\}$

(d) $||\mathbf{x}|| = |x_1|$ (e) $||\mathbf{x}|| = |x_1| - |x_2| + |x_3|$ (f) $||\mathbf{x}|| = \frac{1}{|x_1|} + \frac{1}{|x_2|} + \frac{1}{|x_3|}$

(g) none of the above

2 Computation and Short Answer

4. (5 points) Let

$$S = \text{Span}\{(1,3,1,1)^T, (0,1,0,-1)^T\}.$$

Find a basis for $S^{\perp}$.

5. (5 points) Let $A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$. Without using a calculator and without finding the characteristic polynomial, find all the eigenvalues of A. Show your work.

6. (a) (5 points) Find the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix}.$$

$\lambda_1 =$	$\lambda_2 =$	$\lambda_3 =$

(b) (5 points) For each distinct eigenvalue above, find a basis for the corresponding eigenspace of eigenvectors. Indicate which basis corresponds to which eigenvalue.

(problem continues on next page)

(c) (5 points) Factor A into the product XDX^{-1}, where D is diagonal. If this is not possible, explain why.

$D =$

$X =$

7. Let $\mathbf{u} = (-2, 1, 2)^T$ and $\mathbf{v} = (1, 2, 3)^T$.

(a) (3 points) Find $\mathbf{u} \cdot \mathbf{v}$

(b) (3 points) Find the 2-norm $\|\mathbf{u}\|_2$ (i.e., the Euclidean length of $\mathbf{u}$).

(c) (5 points) Find the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

8. The functions $\cos x$ and $\sin x$ are both unit vectors in $C[-\pi, \pi]$ with inner product defined by

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)\, dx$$

(a) (4 points) Show that $\cos x \perp \sin x$.

(b) (4 points) Determine the value of $\|\cos x + \sin x\|$, where the norm is derived from the inner product in the standard way.

9. (3 points) What are the eigenvalues of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$?

10. (4 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in R^n, the Cauchy-Schwarz inequality says what?

11. (4 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in R^n, the Pythagorean Law says what?

3 Always/Sometimes/Never

12. (12 points) For each of the following statements, if the statement is always true, write “Always” in the blank and write a short justification. If the statement is never true, write “Never” in the blank and write a short justification. If the statement is sometimes true and sometimes false, write “Sometimes” in the blank and give an example when the statement is true and an example when the statement is false.

_____(a) A 2×2 matrix with real entries has real eigenvalues.

_____(b) If A is an $n \times n$ matrix whose eigenvalues are all nonzero, then A is invertible.

_____(c) If A, an $n \times n$ matrix, is diagonalizable, then A has n distinct eigenvalues.

_____(d) Two row-equivalent matrices have the same eigenvalues.

4 The rest of the test

13. (6 points) Let A be an $n \times n$ matrix for which $A^2 = I$. What possible numerical values can the eigenvalues of A be?

14. (6 points) Show that if the $n \times n$ matrices A and B can be diagonalized by the same matrix X, then $AB = BA$. (Hint: Recall that A is diagonalized by the matrix X if $D = X^{-1}AX$, where D is a diagonal matrix.)

15. (6 points) Let $\mathbf{x}$ and $\mathbf{y}$ be vectors in R^n. Show that if $\|\mathbf{x}\| = \|\mathbf{y}\|$, then $\mathbf{x}+\mathbf{y}$ and $\mathbf{x}-\mathbf{y}$ are orthogonal.

16. (6 points) Prove or disprove: If A is an $n \times n$ matrix, then A and A^T have the same eigenvalues.