Problem 1: Define $A_n = \{x \in \mathbb{R} : \frac{-1}{n} \leq x \leq \frac{1}{n}\}$. Find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=1}^{\infty} A_n$.

Solution: For an element x to belong to the set $\bigcap_{n=1}^{\infty} A_n$, it is necessary that x belongs to all the sets $A_1,\ A_2,\ A_3, \ldots$. That is, for all integers $n \geq 1$, we must have $\frac{-1}{n} \leq x \leq \frac{1}{n}$. Since $\lim_{n\to\infty} \frac{-1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n}$, it follows that $x = 0$. That is,

$$\bigcap_{n=1}^{\infty} A_n = \{0\}.$$

For an element x to belong to the set $\bigcup_{n=1}^{\infty} A_n$, it is necessary that x belongs to the set A_n for some $n \geq 1$. Given positive integers m and n, the inequality $\frac{1}{n} \leq \frac{1}{m}$ holds if and only if $m \leq n$. Therefore, it follows that $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$. Thus,

$$\bigcup_{n=1}^{\infty} A_n = A_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}.$$

Problem 2: Derive the quadratic formula. That is, given the quadratic equation

$$f(x) = ax^2 + bx + c,$$

with $a \neq 0$, show that the roots of $f(x)$ are $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$.

Solution: In order to find the roots of the quadratic $ax^2 + bx + c = 0$, we will use the method of completing the square as follows.

$$\begin{aligned}
ax^2 + bx + c &= 0 \\
x^2 + \frac{b}{a}x &= -\frac{c}{a} \\
x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\
x &= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\
&= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Thus, the roots of $f(x)$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$