Math 320 Final Exam Study Guide

1. General Information

- This study guide mostly covers new material since the last midterm, but the final exam will cover all material we have done this semester. To review older material you should review the previous study guides, exams, and homework problems.
- The exam will be in the testing center from Saturday December 15 through Thursday December 20.
- Books, notes, and calculators will not be allowed.
- The study guides are not guaranteed to be exhaustive. Not appearing on the study guides does not mean that something will not appear on the exam.

2. Definitions

Know the definitions discussed in the book and in class, including:

1. Importance, inversion, rejection sampling
2. Hash table
3. $L^2$ inner product
4. Fourier series of a function $f$
5. Piecewise Lipschitz
6. Discrete inner product
7. Discrete Fourier transform
8. Circular convolution
9. Hadamard product
10. Band limited, Nyquist frequency, Nyquist rate
11. Alias
12. Haar father function and mother wavelet

Be able to produce examples and non-examples for these definitions.

3. Theorems/Algorithms You Should Know and Be Able to State and Be Able to Use

Be sure to know the full statement of each theorem, including all hypotheses.

1. Monte Carlo estimate for integrals (uniform and nonuniform distributions)
2. Orthonormality of $\{e^{i\omega k t}\}$ (Theorem 8.1.2) (Which inner product are we using?)
3. Convergence of Fourier series (Theorem 8.1.17)
4. Riemann-Lebesgue
5. Fast Fourier transform
6. Finite convolution theorem
7. Periodic sampling theorem
8. Haar sons form an orthogonal basis for $V_j$ and can be scaled to be orthonormal
9. Haar daughter wavelets form an orthogonal basis for $W_j$, the orthogonal complement of $V_j$ inside $V_{j+1}$
10. The space $V_{j+1}$ can be decomposed into $V_{j+1} = W_j \oplus \perp W_{j-1} \oplus \perp \ldots \oplus \perp W_0 \oplus \perp V_0$.
11. Fast wavelet transform
4. Sample problems

(1) Prove a theorem such as (2), (8), or (10) from the list above.
(2) In what settings would a wavelet decomposition be more useful than a Fourier series? Explain.
(3) Describe the FFT and explain why its temporal complexity is $O(n \log n)$.
(4) Choose four constants $a, b, c, d$. Given a function $f$ that is equal to $a$ on $[0, 0.25)$, equal to $b$ on $[0.25, 0.5)$, equal to $c$ on $[0.5, 0.75)$, equal to $d$ on $[0.75, 1)$, and equal to 0 outside $[0, 1)$, express $f$ in terms of component parts living in the spaces $W_1, W_0, V_0$.
(5) Now assume that $f$ is periodic with period 1 instead of being zero outside the interval $[0, 1)$. Redefine $f$ at the points 0, 0.25, 0.5, 0.75 appropriately and find the exponential Fourier series of $f$.
(6) Choose two vectors $a, b$ in $\mathbb{R}^4$. Compute the convolution $a * b$ directly. Compute the convolution using the DFT, Hadamard product, and inverse DFT instead.