Homework 3, due September 9

(1) Compute $\gcd(455, 1235)$ and $\gcd(2415, 6909)$ by hand. Find integers $x, y, z, w$ such that $455x + 1235y = 65$ and $2415z + 6909w = 63$.

(2) The Fibonacci numbers are given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$. True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.

(3) Let $a$ and $b$ be integers. Using the definition of congruence modulo $n$, prove that $a \equiv b \pmod{n}$ if and only if $a^2 + b^2 \equiv 2ab \pmod{n^2}$.