Homework 36, due December 2

(1) (Problem 4, page 370) The points $(3, \pm 5)$ lie on the elliptic curve $y^2 = x^3 - 2$ defined over the rational numbers. Find another point with rational coordinates that lies on this curve.

(2) (Problem 5, page 370) Show that the point $Q = (2, 3)$ on the curve $y^2 = x^3 + 1$ satisfies $6Q = \infty$. Show that the points $\infty, Q, 2Q, 3Q, 4Q, 5Q$ are distinct.

(3) Consider the point $P = (3, 8)$ on the curve $y^2 = x^3 - 43x + 166$. Compute $2P, 4P,$ and $8P$. 