Homework 3, due September 11

(1) Compute \( \text{gcd}(455, 1235) \) and \( \text{gcd}(2415, 6909) \) by hand. Find integers \( x, y, z, w \) such that 
\[
455x + 1235y = 65 \\
2415z + 6909w = 63.
\]

(2) The Fibonacci numbers are given by \( F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2} \). True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.

(3) Let \( a \) and \( b \) be integers. Using the definition of congruence modulo \( n \), prove that \( a \equiv b \pmod{n} \) if and only if \( a^2 + b^2 \equiv 2ab \pmod{n^2} \).