It has often been a melancholy experience to hear a cadet ask me what the point of a DDS, determinant or continuous function is. I heard similar questions when I was teaching at Rutgers. Sometimes they were asked out of genuine curiosity, but more often they have been out of sheer frustration and disinterest. Indeed, there is nothing more dis-heartening than hearing skepticism about the value of your intellectual activities.

I can relate to this kind of frustration and disinterest. I often questioned the “point” of studying history during my naive high school years. What I can also relate to is that I never asked such questions in classes I excelled regardless of the lack or abundance of the usefulness of the subjects, but only in classes in which I performed poorly, as if the want of utility of the subjects, if any, would provide me with a justifiable excuse for failure.

This kind of skepticism is also unjust for mathematics, I feel. Few of us ever question the “point” of the works of Chaucer or Shakespeare. Granted that their works have affected the development of the English language in very profound ways, but that is not why Shakespeare and Chaucer are remembered as great poets and people make careers out of studying their works. They were great poets because they wrote great poetry and we study their works because of their aesthetic appeal. The importance and value of their works lie in the works themselves, not in any applications or consequences. Such is also true for mathematics. Both from purely academic and pedagogical points of view, the value of mathematics lies in itself.

Often I find the students disinterested in a subject and hence they perform poorly; and their poor performances contribute more to their disinterests. Thus a vicious cycle is formed and the skepticisms mentioned earlier is but manifestation of the disinterest. What I am disposed to write in this essay are some ideas about the curriculum I have had and things I have experimented on breaking this cycle, from the point of view of interest.

I genuinely find mathematics interesting and beautiful. I cannot do better than quoting Godfrey Harold Hardy [1] in this regard.

His[a mathematician’s] subject is the most curious of all – there is none in which truth plays such odd pranks. It has the most elaborate and most fascinating technique, and gives unrivalled openings for the display of sheer professional skill.

Indeed, mathematics has never failed in presenting marvels to its patrons. But somehow this interest is not felt by all my students. Hence I believe that it is one of my first duties as a college professor to impart some of this appreciation for mathematics toward my students.

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I was surprised to discover, during the third summer of my graduate studies, that Rutgers has mathematics course titled, Mathematics for the Liberal Arts. It is a freshmen course and instead of having the students study the standard calculus or pre-calculus, it teaches elementary graph theory, probability, number theory and combinatorics. It teaches concepts such as trees, Euler paths, modular arithmetics, and the pigeon-hole principle. The course requires almost no prerequisite whatsoever and can be taken by essentially anyone. It also touches upon problems such as decomposition of the Congressional districts and computing the consumer price index. Relating this revelation to my fellow graduate students at the time, I found consensus among my peers that students would enjoy taking such a course much more than calculus or pre-calculus and similar feedbacks were gotten from the students. I felt that a course like that was genuinely much more interesting than any course in elementary university mathematics I have taught. All good mathematics has the characteristics of unexpectedness as different ideas are being connected and inevitability as mathematical conclusions are inescapable. Elementary combinatorics, graph theory and number theory often offer something that calculus and the like do not: economy. When problems are viewed from the right angle, one line of attack suffices to solve them. Indeed, we all have the experience of seeing our students dreading through seeing a lengthy calculus problem being solved on the board.

Pedagogically, I believe, and I would imagine that many people would agree with me, that mathematics is taught in universities because there are many crucial skills that an educated person should possess. Among these skills, perhaps the most important ones would be the abilities to reason and to present one’s ideas in a clear and understandable fashion. No subject illustrates and imparts these skills toward the students better than mathematics. A calculus student will probably not remember the chain rule of differentiation years after his graduation, but if teaching and learning have been done properly, he would be in the possession of something a lot more valuable: the skills to solve complicated problems with reasoning and present the solution clearly, id est knowing how to make known the unknown. If those are our goals, then it matters not at all in what context we impart these skills. The reasoning that is required to show the infinitude of prime numbers or the number of vertices in a tree is always one more than the number of its edges has the same pedagogical value as any problem such as computing the balance of an IRA account using DDS or computing the volume of a irregularly shaped lake using a triple integral. Quoting again from [1],

For what is useful above all is technique, and mathematical technique is taught mainly through pure mathematics.

I am certainly not suggesting the elimination of the calculus sequence at the college level, as such courses must be taken by all who study science and engineering. But rather, I am suggesting, as I dread to see how many of my students have grown disgusted with calculus and the like which lead them to ultimately loath mathematics altogether, that we should have a course that teaches mathematics that could be considered interesting by students, the kind of mathematics that is understandable to all trained intelligence and that would raise one’s eyebrow when he sees it and I believe that teaching the liberal art students elementary combinatorics, graph theory and number theory offers precisely that. Such a course need not go deep into mathematics, but ought to cover many various topics. There is no escape for a man of science or engineering from studying calculus and the like, but he ought to be somewhat mathematically inclined in the first place. However, for someone who has greater need in studying the spirit of mathematics rather than its contents,
perhaps calculus is not the only avenue for success.

The following is something I have done before. Many of my students are perhaps too concerned only with their grades to notice or appreciate the beauty of mathematics that is presented to them. There are also others who come to mathematics classes with negative attitudes, thinking mathematics is boring and useless and are simply taking the courses because they have to. The same can be said about many of the cadets at USMA I have taught. There is little I can do during class about this as there is much material to be covered. But to rectify the situation, I devised a few years ago the extra credit of having the students read Hardy’s *A Mathematician’s Apology* [1] and write a book report. Hardy’s essay gives examples and philosophies of the beauty of mathematics perfectly, and it can be well-understood by non-mathematicians such as my students. More importantly, it contains two results in elementary mathematics that I believe most would definitely consider interesting: Euclid’s proof of infinitude of the prime numbers and Pythagoras’s proof of the irrationality of $\sqrt{2}$. Neither requires more knowledge of mathematics than the Euclidean algorithm. My hope was that the assignment can generate some interest in mathematics in the minds of my students in showing them some of the best results our subject has to offer. The outcome has been overwhelmingly positive. Not only was there no pain at all to read the book reports and have the extra grading, but I found much pleasure in the reading. I remember one of my students wrote to me after reading the *Apology* that after his college years reading Hardy’s *Apology* would likely be the only thing he remembers about his calculus course. Also there were students who expressed strong opinions of disagreement with Hardy and wrote passionately to that end; and many agreed with Hardy and started to compare mathematics to poetry and the arts. One student enjoyed the experience so much that she bought copies of the *Apology* to share with her family members*. This almost brought tears to my eyes. There is nothing more flattering to a craftsman than having him find many people interested in his trade. I was very pleased that many of my students read the essay and were indeed moved by it. Many of them did seem to have a better understanding of the beauty of mathematics of which they would otherwise be unaware. In brief, I was very happy with the outcome of this assignment and have been giving it to my classes ever since.

The mathematics in both the *Apology* and the Rutgers course are easily accessible by anyone, and what is also true is that both offer the best mathematics has to offer which is the best chance we have at such “enterprises of great pith and moment”([2], *Hamlet*, III, i, 88) as imparting the beauty of mathematics and generating interest in mathematics. For many of our students, we need to break the vicious cycle somewhere.

**References**


**Department of Mathematical Sciences**  
**United States Military Academy**  
**West Point, New York 10996 USA**  
**Email Address: Liangyi.Zhao@usma.edu**

*The book is rather cheap.*