Review for Math 334 – Final

Chapter 1

Important skills:
1. Match a direction field to an ODE (Ex. 2, p. 3)
2. Know what an initial value problem is and how to show a given function is a solution to one. (Ex 2, p. 13)
3. Know how to classify differential equations by order and linearity. (p. 20-21)

Chapter 2

Important skills:
1. Be able to determine if a first order differential equation is linear or nonlinear. Equation (3) on page 21 gives the form for a linear ODE.
2. If the differential equation is linear, compute the integrating factor, and then the general solution. (Ex. 4, p. 38)
3. If the differential equation is nonlinear, then is it separable? If it is, then you will need to compute two integrals.
4. You will need to know basic integration techniques. For example, various substitutions, integration by parts, and partial fractions will all be utilized.
5. If the equation is nonlinear and not separable is it exact? If so, solve it using the method in Section 2.6. (Ex. 2, p. 97)
6. What happens to equations as time goes to infinity? Understand stability, asymptotic stability and instability.
7. Understand the three steps in the process of mathematical modeling: construction of the model, analysis of the model, and comparison with experiment or observation. (Ex. 3, p. 54)
8. Determine the existence and uniqueness of solutions to differential equations. (Ex. 2, p. 71)
9. Know how to recognize autonomous equations, and utilize the direction field to represent solutions to them. Be able to determine asymptotically stable, semi-stable, and unstable equilibrium solutions. (Ex. 1, p. 83)
10. Mixing problems, compound interest, motion in a gravitational field, radioactive carbon dating, cooling.

Chapter 3

Important skills:
1. Be able to determine if a second order differential equation is linear or nonlinear, homogeneous, or nonhomogeneous. (If it can be put into the form given by Equation (3) in page 138, it is linear.)
2. Can you recognize a homogeneous equation with constant coefficients, and derive the characteristic equation? (Ex. 3, o. 149) This equation will be quadratic, so
know the quadratic formula, the types of solutions one gets: real and distinct, repeated, and complex conjugate. These three cases will be crucial to the types of solutions one gets to constant coefficient homogeneous differential equations.

3. Be able to write down fundamental solution sets to homogeneous equations. This means find two solutions. (Ex. 3, p. 149)

4. What are the fundamental solution sets for each of the three cases of roots when solving constant coefficient equations? The summary is on p. 170. (Ex. 3, p. 149; Ex. 2, p. 169; Ex. 3, p. 162)

5. Solutions to second order nonhomogeneous equations have two components. There is the homogeneous solution, and particular or nonhomogeneous solution. (Thm. 3.5.2, p. 174) To find particular solutions you must know the method of undetermined coefficients, and variation of parameters. (Ex. 4, p. 178; Ex. 1, p. 185)

Chapter 4

Important skills:
1. The methods for solving higher order linear differential equations are extremely similar to those of the last chapter. The general solution to an nth order homogeneous linear differential equation is obtained by linearly combining n linearly independent solutions. (Eq. (5), p. 220)
2. The generalization of the Wronskian is given on page 221. It is used as in the last Chapter to show the linear independence of functions, and in particular, homogeneous solutions.
3. For the situation where there are constant coefficients, you should be able to derive the characteristic polynomial, and the characteristic equation, in this case of nth order. Depending on the types of roots you get to this equation you will have solution sets containing functions similar to those in the second order case. (Ex. 2-4, p. 229-231)

Chapter 5

Important skills:
1. Review power series, how to shift the index of summation, (Ex. 3, p. 247) and tests for convergence.
2. Know how to find the interval of convergence for a power series. (Ex. 2, p. 245)
3. For ordinary points, Eq. (3) on page 251 gives the form of the solution. Be able to derive the recursion relation, as in Example 1. If the recursion relation can be solved, one obtains the two solutions of the homogeneous problem. (Ex. 1, p. 251)
4. The method described in the second paragraph on page 244 can be used to find the first several terms in each of the homogeneous solutions.
5. Be able to determine lower bounds on the radius of convergence of the series solutions. (Ex. 4, p. 264)
Chapter 6

*Important skills:*

1. The Laplace transform is defined through an improper integral. You must be comfortable evaluating improper integrals. Hence you should review this topic in any calculus book.
2. Be able to calculate the Laplace transform of the basic functions given in the table on p. 317. (You will be given this table.)
3. Be able to compute inverse transform functions. You will need to be able to use partial fractions.
5. Understand the unit step function, as well as, the unit impulse function, and how to use them in transforming and inverse transforming functions.
6. The process of the Laplace transform is as follows: i. Given a differential equation, one transforms the equation using the initial conditions. ii. Solve the equation for $Y(s)$. iii. Compute $y(t)$ by finding the inverse of $Y(s)$.

Chapter 7

*Important skills:*

1. Find the solution set of linear algebraic equations (Ex. 1, p. 375)
2. Find the eigenvalues and eigenvectors of a matrix (Ex. 5, p. 381)
3. Sketch a direction field for a 2x2 system of linear ODE’s (Ex. 2, p. 394)
4. Find the general solution of a system of linear ODE’s with distinct eigenvalues either real (Ex. 3, p. 397), complex (Ex. 1, p. 401), or repeated roots (Ex. 2, p. 423)
5. Find the fundamental matrix for a system of linear ODE’s. (Ex. 1 and 2, p. 414-415)

Chapter 9

*Important skills:*

1. Be able to determine the phase plane and phase portraits of 2 by 2 linear systems. The solutions will depend on the eigenvalues. Pages 486-493 cover the five important cases. Table 9.1.1 on page 494 summarizes the eigenvalue results.
2. Know how to determine whether a system of ODE’s is locally linear (Ex. 1 and 2, p. 510).
3. Be able to determine the linear system associated with the almost linear system (Ex. 3, p. 512)
4. Sketch phase portraits for competing species (Ex. 1 or 2, p. 521 and p. 524).
5. Sketch phase portraits for predator-prey systems (Ex. 1, p. 534).